Definations:-

Matrix:- • The rectangular (or) square arrangement of real (or) Complex no's is Called Matrix.

• The Square brackets [ ] are used to represent the matrix.

• Eg:- $A = \begin{bmatrix} 2 & -i \\ 0 & 1 \end{bmatrix}_{2\times2}$, $A = \begin{bmatrix} 2i & -i \\ 0 & 1 \\ -2 & 3 \end{bmatrix}_{3\times2}$

Square Matrix:- • A matrix in which rows and Colouns are equal is Called square matrix.

• eg:- $A = \begin{bmatrix} 2 & -i \\ 0 & 1 \end{bmatrix}_{2\times2}$, $A = \begin{bmatrix} -1 & 1 & -1 \\ 2 & 3 & -1 \\ 3 & 1 & -4 \end{bmatrix}_{3\times3}$

Retangular Matrix:- • A matrix in which rows and Colouns are not equal is Called retangular matrix.

• Eg:- $A = \begin{bmatrix} 2i & -i \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$

Identity Matrix:-
(or)
Unit Matrix

• A matrix in which the elements of diagonals are 1 & rest of the elements are 0 is Called identity matrix. & it is denoted by I.

• eg:- $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$

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Note:- Trace of a matrix.

⟹ The trace of a matrix is denoted by trace of A and it is given by the sum of elements of diagonal.

⟹ eg:- $A = \begin{bmatrix} -1 & 1 & -1 \\ 2 & 3 & -1 \\ 3 & 1 & -4 \end{bmatrix}$

$= -1 + 3 - 4$

$= -5 + 3 = 2$

General representation of Matrix:- $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

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Upper triangular matrix:-

⟹ A matrix which satisfies the condition $[a_{ij}] = 0$ when $i > j$ is called upper triangular matrix.

[or]

⟹ A matrix in which below diagonal elements are zero is called upper triangular matrix.

⟹ eg:- Ex:- $\begin{bmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{bmatrix}$

Lower triangular matrix:-

⟹ A matrix which satisfies the condition $[a_{ij}] = 0$ when $i < j$ is called lower triangular matrix.

⟹ eg:- Ex:- $\begin{bmatrix} * & 0 & 0 \\ * & * & 0 \\ * & * & * \end{bmatrix}$

Symmetric matrix:- A matrix which satisfies the following conditions is called symmetric matrix.

[i] $a_{11} \neq a_{22} \neq a_{33}$

[ii] $a_{12} = a_{21}$

$a_{32} = a_{23}$

$a_{13} = a_{31}$

[or]

⇒. A matrix which satisfies the condition $A^T = A$ then A is called symmetric matrix.

Eg:- $A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & -1 & -1 \\ 5 & -1 & 0 \end{bmatrix}$

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Skew- symmetric matrix:- A ~~matr~~ matrix which satisfies the following conditions is called skew symmetric matrix.

[i] $a_{11} = a_{22} = a_{33}$

[ii] $a_{12} + a_{21} = 0$

$a_{32} + a_{23} = 0$

$a_{13} + a_{31} = 0$

[or]

⇒. A matrix which satisfies the condition $A^T = -A$ then A is called skew symmetric matrix.

Eg:- $A = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 2 \\ -3 & -2 & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$

$-A = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$

$\therefore A^T = -A$

$\therefore$ A is skew symmetric matrix.

Rank

Determinents:-

1) $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -2 \\ 1 & 2 & 4 \end{bmatrix}$

$|A| = 3(8+4) - 1(4+2) + 2(2-2)$

$= 3(12) - 1(6)$

$= 36 - 6$ $\Rightarrow$ Rank of A is 3

$|A| = 30 \neq 0$ $\qquad \rho(A) = 3.$

2) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix}$

$= +1(0+0) - 2(0-0) + 3(0-0)$ $\Rightarrow$ Rank of A is 1

$|A| = 0$ $\qquad \rho(A) = 1$

3) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= 1(1-1) - 1(1-1) + 1(1-1)$ $\Rightarrow$ Rank of A is 1

$|A| = 0$

$\rho(A) = 1$

4) $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$

$= 3(4-2) - 2(8-6) + 1(4-6)$

$= 3(2) - 2(2) + 1(-2)$

$= 6 - 4 - 2$

$= 0$

$A_1 = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}_{\boxed{2\times 2}}$ $\Rightarrow$ Rank of A is 2

$= 3 - 4$ $\qquad \rho(A) = 2$

$= -1 \neq 0.$

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Rank of the matrix:-

⇒ The rank of the matrix is defined as the no. of non-zero rows in that matrix.

⇒ The rank of the matrix is denoted by $\rho(A)$.

Case 1:- If $|A| \neq 0$ then no. of non-zero rows is the rank.

Case 2:- If ~~|A|≠0~~ $|A| = 0$ and if there exist atleast one submatrix whose determinant is not zero then rank of the matrix is equal to order of sub matrix.

Case 3:- If $|A| = 0$ and determinant of every sub-matrix is also zero then rank is 1.

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Rank of matrix:

⇒ Let 'A' represent a non-zero matrix, a number 'r' is said to be rank of the matrix A if:-

1) Every $(r+1)^{th}$ manner order of A is zero.
2) Atleast 1 manner of order 'r' which is non-zero.

Echelon form:-

⇒ A $m \times n$ matrix is said to be in echelon form if:-

1) All zero rows (or) any zero row occurs below the non-zero row.
2) The no. of zero's before the first non-zero element in the row are in increasing order.

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3) The ~~fs~~ first non-zero element in every row is unity [optional].

1Q) Reduce the matrix into echelon form and find its rank.

(or)

Find the rank of the matrix.

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{bmatrix}$$

Sol) Given;

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

$R_2 \rightarrow 3R_2 - 2R_1$, $R_3 \rightarrow R_3 - 2R_1$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{bmatrix}$$

$R_3 \rightarrow \dfrac{R_3}{-2}$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_2$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

No. of Non-zero rows = 2

$\rho(A) = 2$

(or)

sol) Given;

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

$= 3(4-2) - 2(8-6) + 1(4-6)$

$= 3(2) - 2(2) + 1(-2)$

$= 6 - 6$

$= 0$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$$

$= 4 - 2 = 2 \neq 0$

$\rho(A) = 2.$

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2Q) $A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7 \end{bmatrix}$

sol) Given;

$$A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 2R_1$

$R_3 \rightarrow R_3 + 3R_1$

$$A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 0 & 5 & 1 & 16 \\ 0 & 8 & 4 & 31 \end{bmatrix}$$

$R_3 \rightarrow 5R_3 - 8R_2$

$$A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 0 & 5 & 1 & 16 \\ 0 & 0 & 12 & 27 \end{bmatrix}$$

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$R_3 \to \frac{R_3}{3}$

$$A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 0 & 5 & 1 & 16 \\ 0 & 0 & 4 & 9 \end{bmatrix}$$

$\therefore$ The No. of non-zero rows

$\rho(A) = 3$

Q3) Reduce the matrix into Echelon form (or) Find the rank of matrix.

$$A = \begin{bmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{bmatrix}$$

Sol)

$|A| = -1(18-1) - 0(0) + 6(3+30)$

$= -1(17) + 6(33)$

$= -181 \neq 0$

$\rho(A) = 3$

(or)

$$A = \begin{bmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{bmatrix}$$

$R_2 \to R_2 + 3R_1$

$R_3 \to R_3 - 5R_1$

$$A = \begin{bmatrix} -1 & 0 & 6 \\ 0 & 6 & 19 \\ 0 & 1 & -27 \end{bmatrix}$$

$R_3 \to 6R_3 - R_2$

$$A = \begin{bmatrix} -1 & 0 & 6 \\ 0 & 6 & 19 \\ 0 & 0 & -181 \end{bmatrix}$$

$\therefore$ No. of non-zero rows $= 3$

$\rho(A) = 3.$

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Solutions of linear System:-

Consider the System of Eqn's are:-

$a_1x+b_1y+c_1z=d_1$

$a_2x+b_2y+c_2z=d_2$

$a_3x+b_3y+c_3z=d_3$

①. The matrix form is

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$AX=B.$

②. The Augmented matrix is

$$[A:B] = \left[\begin{array}{ccc:c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$$

Case(i):- $\rho(A) = \rho(A:B) = n$

Then the system is Consistent & has unique Sol'n's

Case(ii):- $\rho(A) = \rho(A:B) < n$

Then the System is Consistent & has many Sol'n's

Case(iii):- $\rho(A) \neq \rho(A:B)$

Then the system is Inconsistent & No solution

Q) Solve the system of Eqn's:-

$$x - 2y + 4z = 12$$
$$2x - y + 5z = 18$$
$$-x + 3y - 3z = -8$$

Sol) The Augmented matrix is:-

$$[A:B] = \left[\begin{array}{ccc|c} 1 & -2 & 4 & 12 \\ 2 & -1 & 5 & 18 \\ -1 & 3 & -3 & -8 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 + R_1$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 4 & 12 \\ 0 & 3 & -3 & -6 \\ 0 & 1 & 1 & 4 \end{array}\right]$$

$R_2 \to \frac{R_2}{3}$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & -2 & 4 & 12 \\ 0 & 1 & -1 & -2 \\ 0 & 1 & 1 & 4 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & -2 & 4 & 12 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 2 & 6 \end{array}\right]$$

$\rho(A) = 3 \qquad \rho(A:B) = 3 \qquad n = 3$

$\rho(A) = \rho(A:B) = n$

$\therefore$ The system is consistent & It has unique soln's.

The sol^n is

$2z = 6$

$\boxed{z=3}$

$y - z = -2$

$y - 3 = -2$

$\boxed{y = 1}$

$x - 2y + 4z = 12$

$x - 2 + 12 = 12$

$\boxed{x = 2}$

8) Solve the system of $eq^{n's}$:- If consistent find complete sol^n.

$x + y - 3z = 1$

$2x + y - 4z = 0$

$-3x + 2y - z = 7.$

Sol) The Augmented matrix is:-

$$[A:B] = \left[\begin{array}{ccc:c} 1 & +1 & -3 & 1 \\ 2 & 1 & -4 & 0 \\ -3 & 2 & -1 & 7 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 + 3R_1$

$$= \left[\begin{array}{ccc:c} 1 & -1 & -3 & 1 \\ 0 & 3 & 2 & -2 \\ 0 & 8\ -1 & -10 & 10 \end{array}\right] = \left[\begin{array}{ccc:c} 1 & 1 & -3 & 1 \\ 0 & -1 & 2 & -2 \\ 0 & 5 & -10 & 10 \end{array}\right]$$

$R_3 \to \dfrac{R_3}{5}$

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & -3 & 1 \\ 0 & -1 & 2 & -2 \\ 0 & 1 & -2 & 2 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$= \left[\begin{array}{ccc:c} 1 & 1 & -3 & 1 \\ 0 & -1 & 2 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = \rho(A:B) < n$

∴ The system is consisted but it has infinite sol^n.

$x+y-3z=1$

$-y+2z=-2$

$-y+2k=-2$

$\cancel{(-y+2)}(-y = -2-2k)$

$\boxed{y = 2+2k}$

Let $\boxed{z=k}$

$x+2+2k-3k=1$

$x-k = 1-2$

$x-k=-1$

$x=-1+k$

$\boxed{x = k-1}$

Q) ~~Bx~~ Solve the system of eq$^{n's}$. It consistant find complete sol^n.

$3x-4y-z=3$

$2x-3y+z=1$

$x-2y+3z=2$

Sol) The Augmented matrix is

$$[A:B] = \left[\begin{array}{ccc:c} 3 & -4 & -1 & 3 \\ 2 & 3 & 1 & 1 \\ 1 & -2 & 3 & 2 \end{array}\right]$$

$R_2 \to 3R_2 - 2R_1$

$R_3 \to 3R_3 - R_1$

$$= \left[\begin{array}{ccc:c} 3 & -4 & -1 & 3 \\ 0 & -1 & 5 & -3 \\ 0 & -2 & 10 & 3 \end{array}\right]$$

$R_3 \to R_3 - 2R_2$

$$[A:B] = \left[\begin{array}{ccc:c} 3 & -4 & -1 & 3 \\ 0 & -1 & 5 & -3 \\ 0 & 0 & 0 & 9 \end{array}\right]$$

$\rho(A) = 2 \qquad \rho(A:B) = 3$

$\rho(A) \neq \rho(A:B)$

$\therefore$ The System is inconsistent & has No Solution.

Q) Solve the System of eqn's.

$x + y + z + t = 4$

$x - z + 2t = 2$

$y + z - 3t = -1$

$x + 2y - z + t = 3$.

Sol) The Augmented matrix

$$[A:B] = \left[\begin{array}{cccc:c} 1 & 1 & 1 & 1 & 4 \\ 1 & 0 & -1 & 2 & 2 \\ 0 & 1 & 1 & -3 & -1 \\ 1 & 2 & -1 & 1 & 3 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$R_4 \to R_4 - R_1$

$$[A:B] = \left[\begin{array}{cccc:c} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & 1 & -2 \\ 0 & 1 & 1 & -3 & -1 \\ 0 & 1 & -2 & 0 & -1 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$R_4 \to R_4 + R_2$

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$$= \left[\begin{array}{cccc:c} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & 1 & -2 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & -4 & 1 & -3 \end{array}\right]$$

$R_4 \to R_4 - 4R_3$

$$= \left[\begin{array}{cccc:c} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & 1 & -2 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 9 & 9 \end{array}\right]$$

$\rho(A) = 4 \qquad \rho(A{:}B) = 4$

$9t = 9 \Rightarrow \boxed{t = 1}$

$-z - 2t = -3$

$-z - 2 = -3$

$-z = -3 + 2$

$\boxed{z = -1}$

$-y - 2z + t = -2$

$-y - \cancel{2} + 1 = -\cancel{2}$

$\boxed{y = 1}$

$x + y + z + t = 4$

$\boxed{x = 1}$

H/w:-

8) $2x - y - z = 2$

$x + 2y + z = 2$

$4x - 7y - 5z = 2$

$2(2)+1$
$2(1)+1$
$2(2)-2$

$-7-2(-1)$
$-7+2$

$-5-2(-1)$
$-5+2$

$2-2(2)$
-2

Sol) The augmented matrix is

$$[A:B] = \left[\begin{array}{ccc:c} 2 & -1 & -1 & 2 \\ 1 & 2 & 1 & 2 \\ 4 & -7 & -5 & 2 \end{array}\right]$$

$R_2 \to 2R_2 - R_1$

$R_3 \to R_3 - 2R_1$

$$[A:B] = \left[\begin{array}{ccc:c} 2 & -1 & -1 & 2 \\ 0 & 5 & 3 & 2 \\ 0 & -5 & -3 & -2 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$[A:B] = \left[\begin{array}{ccc:c} 2 & -1 & -1 & 2 \\ 0 & 5 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$; $\rho(A:B) = 2$

$\rho(A) = \rho(A:B) < n$

$2x - y - z = 2$

$5y + 3z = 2$

$\boxed{\text{Let } z = k}$

$5y + 3k = 2$

$5y = 2 - 3k$

$\boxed{y = \frac{2-3k}{5}}$

$2x - \left(\frac{2-3k}{5}\right) - k = 2$

$\cancel{10x - 2 - 3k - 5k = 2}$

$\cancel{10x - 8k}$

$10x - 2 + 3k - 5k = 10$

$10x - 2K = 10+2$

$10x - 2K = 12$

$10x = 12 + 2K$

$\overset{5}{\cancel{10}}x = 2(6+K)$

$5x = 6+K$

$$\boxed{x = \frac{6+K}{5}}$$

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Q) Find the Values of a & b ~~A&B~~ for which the System of eqⁿ is

$x+y+z=6$

$x+2y+3z=10$

$x+2y+az=b$

have:-

(i) Unique solⁿ.
(ii) Infinite solⁿ.
(iii) No solⁿ.

Sol) The matrix form is

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ b \end{bmatrix}$$

Augmented matrix.

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 1 & 2 & a & : & b \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 1 & a-1 & : & b-6 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & a-3 & b-10 \end{array}\right]$$

Case(i) :- $a = 3$; $b = 10$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$; $\rho(A:B) = 2$

$\rho(A) = \rho(A:B) < n$

Infinite solⁿ.

Case(ii) :- $a \neq 3$ $(a = 2)$; $b = 10$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \end{array}\right]$$

$\rho(A) = 3$; $\rho(A:B) = 3$

$\rho(A) = \rho(A:B) = n$

Unique solⁿ.

Case(iii) :- $a = 3$; $b \neq 10$ $(b = 9)$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

$\rho(A) = 2$; $\rho(A:B) = 3$

$\rho(A) \neq \rho(A:B)$

No solⁿ.

9) $x - 3y - 8z + 10 = 0$

$3x + y - 4z = 0$

$2x + 5y + 6z - 13 = 0.$

$1 - 3(-3)$
$-4 - 3(-8)$
$5 - 2(-3)$
$6 - 2(-8)$
$13 - 2(-10)$

Sol)

Augmented matrix $\Rightarrow$ $x - 3y - 8z = -10$

$3x + y - 4z = 0$

$2x + 5y + 6z = 13$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & -3 & -8 & -10 \\ 3 & 1 & -4 & 0 \\ 2 & 5 & 6 & +13 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & -3 & -8 & -10 \\ 0 & 10 & 20 & 30 \\ 0 & 11 & 22 & 33 \end{array}\right]$$

~~$R_3 \to R_3$~~

$R_3 \to \frac{R_3}{11}$; $R_2 \to \frac{R_2}{10}$

$$(A:B) = \left[\begin{array}{ccc|c} 1 & -3 & -8 & -10 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & -3 & -8 & -10 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$; $\rho(A:B) = 2$

$\rho(A) = \rho(A:B) < n$

Infinite Sol^n

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Q) $x+y+z=6$

$2x+3y-2z=2$

$5x+y+2z=13$

Sol) The matrix form:-

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -2 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 13 \end{bmatrix}$$

Augmented matrix

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 2 & 3 & -2 & 2 \\ 5 & 1 & 2 & 13 \end{array}\right]$$

$\Rightarrow$ $R_2 \to R_2 - 2R_1$ $\quad R_3 \to R_3 - 5R_1$

$3 - 2(1)$

$-2 - 2(1)$

$2 - 2(6)$

$13 - 5(6)$

$13 - 30$

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & -4 & -10 \\ 0 & -4 & -3 & -17 \end{array}\right]$$

$R_3 \to R_3 + 4R_2$

$-3 + 4(-4)$

$-3 - 16$

$-17 + 4(-10)$

$-17 + 40$

$$= \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & -4 & -10 \\ 0 & 0 & -19 & -57 \end{array}\right]$$

$\Rightarrow$ $\rho(A) = 3$; $\rho(A:B) = 3$

$\rho(A) = \rho(A:B) = 3$

Unique sol^n.

2024 Q.

Q) Find all the values of 'K' such that the rank of matrix $A = \begin{bmatrix} K & -1 & 0 & 0 \\ 0 & K & -1 & 0 \\ 0 & 0 & K & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix}$ is equal to '3'.

Sol) Given $\rho(A) = 3$

$|A| = 0$

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$$\begin{vmatrix} K & -1 & 0 & 0 \\ 0 & K & -1 & 0 \\ 0 & 0 & K & -1 \\ -6 & 11 & -6 & 1 \end{vmatrix} = 0$$

$$K \begin{vmatrix} K & -1 & 0 \\ 0 & K & -1 \\ 11 & -6 & 1 \end{vmatrix} + 1 \begin{vmatrix} 0 & -1 & 0 \\ 0 & K & -1 \\ -6 & -6 & 1 \end{vmatrix} + 0 - 0 = 0$$

$$K[K(K-6)+1(0+11)] + 1[0(0)+1(0-6)] = 0$$

$$K[K^2-6K+11]-6 = 0$$

$$K^3-6K^2+11K-6=0$$

Let $\boxed{K=1}$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ & 0 & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$$K^2-5K+6=0$$

$$K^2-2K-3K+6=0$$

$$K(K-2)-3(K-2)=0$$

$\boxed{K=1}$ $\boxed{K=3}$ $\boxed{K=2}$

Q) Find the rank of matrix

Q1) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

2Q) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

3Q) $\begin{bmatrix} 3 & 1 & 4 & 6 \\ 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 1 & 1 & 2 & 2 \end{bmatrix}$

4Q) $\begin{bmatrix} 2 & -1 & 3 & 4 \\ 6 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$

5Q) $\begin{bmatrix} 2 & -1 & 3 & 2 \\ -4 & 0 & 3 & 5 \\ 7 & 2 & 1 & 1 \end{bmatrix}$

Sol 1)

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \Leftrightarrow R_3$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3.$

2) $A = \begin{bmatrix} 3 & 1 & 4 & 6 \\ 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 1 & 1 & 2 & 2 \end{bmatrix}$

Sol)

$R_2 \to 3R_2 - 2R_1$

$R_3 \to 3R_3 - 4R_1$

$R_4 \to 3R_4 - R_1$

$3 - 2$

$6 - 8$

$$A = \begin{bmatrix} 3 & 1 & 4 & 6 \\ 0 & 1 & -2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 2 & 2 & 4 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$= \begin{bmatrix} 3 & 1 & 4 & 6 \\ 0 & 1 & -2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$-1 - 2(-2)$

$-1+4 = 3$

$$= \begin{bmatrix} 3 & 1 & 4 & 6 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$= \begin{bmatrix} 3 & 1 & 4 & 6 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3.$

11/09/24.

Q) If $A = \begin{bmatrix} 3 & x & 6 \\ 5 & 2 & 4 \\ 6 & 4 & 8 \end{bmatrix}$ is Symmetric. Find x.

Sol) Given 'A' is Symmetric

$\Rightarrow A^T = A$

$= \begin{bmatrix} 3 & 5 & 6 \\ x & 2 & 4 \\ 6 & 4 & 8 \end{bmatrix} = \begin{bmatrix} 3 & x & 6 \\ 5 & 2 & 4 \\ 6 & 4 & 8 \end{bmatrix}$

$\boxed{x = 5}$

Q) If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ is Skew-symmetric, find x.

Sol) Given 'A' is Skew-symmetric.

$\Rightarrow A^T = -A.$

$= \begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & x \\ 1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & 2 \\ 1 & -x & 0 \end{bmatrix}$

$\boxed{x = 2}$

Q) Find rank of $\begin{bmatrix} 4 & -2 & 2 \\ 5 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$

Sol) Let $A = \begin{bmatrix} 4 & -2 & 2 \\ 5 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$

$|A| = +4(3-4) + 2(5-2) + 2(20-6)$

$= 4(-1) + 2(3) + 2(14)$

$= -4 + 6 + 28$

$|A| = 30 \neq 0$

$\rho(A) = 3$

By Using Eclon form:-

Sol)

$$A = \begin{bmatrix} 4 & -2 & 2 \\ 5 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$$

$R_1 \to \frac{R_1}{2}$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 5 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$$

$R_2 \to 2R_2 - 5R_1$

$R_3 \to R_3 - R_1$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 11 & -3 \\ 0 & 5 & 0 \end{bmatrix}$$

$R_3 \to 11R_3 - 5R_2$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 11 & -3 \\ 0 & 0 & 15 \end{bmatrix}$$

$\rho(A) = 3$

19/09/24

* Eigon Values and Eigen Vectors:-

Eigen Values:- The roots of Characteristic eqⁿ $|A-\lambda I|=0$ is called Eigen Values. / Latent roots / Characteristic roots.

* Non - Repeated Eigon values:-

Q) Find the Eigon Values and corresponding Eigen vectors for the matrix $A=\begin{bmatrix}3&1&4\\0&2&6\\0&0&5\end{bmatrix}$

Sol)
$$A=\begin{bmatrix}3&1&4\\0&2&6\\0&0&5\end{bmatrix}$$

The characteristic eqⁿ is

$$|A-\lambda I|=0 \Rightarrow \begin{vmatrix}3-\lambda&1&4\\0&2-\lambda&6\\0&0&5-\lambda\end{vmatrix}=0$$

$$\Rightarrow +(3-\lambda)\left[(2-\lambda)(5-\lambda)\right]+0+0$$

$$\lambda=2,3,5$$

Case(i):- $\lambda=2$ Let $X_1=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$

$$\begin{bmatrix}1&1&4\\0&0&6\\0&0&3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$x_1+x_2+4x_3=0$ — (1)

$0x_1+0x_2+6x_3=0$ — (2)

$0x_1+0x_2+3x_3=0$ — (3)

Solving (1) & (2)

$x_1+x_2+4x_3=0$

$0x_1+0x_2+6x_3=0$

$x_1 + x_2 + 4x_3 = 0$

$0x_1 + 0x_2 + 6x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 1 & 4 \\ 0 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 4 \\ 0 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}}$$

$$\frac{x_1}{6} = \frac{-x_2}{6} = \frac{x_3}{0}$$

$$X_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

Case (iii):- $\lambda = 3$ Let $X_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 & 4 \\ 0 & -1 & 6 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0x_1 + x_2 + 4x_3 = 0$

$0x_1 - x_2 + 6x_3 = 0$

$$\frac{x_1}{6+4} = \frac{-x_2}{0-0} = \frac{x_3}{0-0}$$

$$\frac{x_1}{10} = \frac{x_2}{0} = \frac{x_3}{0}$$

$$X_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

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Case (iii):- Let $\lambda = 5$ Let $X_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} -2 & 1 & 4 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + x_2 + 4x_3 = 0$$
$$x_1 - 3x_2 + 6x_3 = 0$$

$$\frac{x_1}{6+12} = \frac{+x_2}{+12} = \frac{x_3}{6}$$

$$\frac{x_1}{\cancel{18}\ 3} = \frac{x_2}{\cancel{12}\ 2} = \frac{x_3}{\cancel{6}\ 1}$$

$$x_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

9)

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Sol) The Char. eq^n is

$$|A - \lambda I| = 0 \Rightarrow \begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ -2 & -4 & 3-\lambda \end{bmatrix}$$

$$\Rightarrow +(8-\lambda)\left[(7-\lambda)(3-\lambda) - 16\right] + 6\left[-6(3-\lambda) - 8\right] + 2\left[24 - 2(7-\lambda)\right] = 0.$$

$$\Rightarrow 8-\lambda\left[21 - 7\lambda - 3\lambda + \lambda^2 - 16\right] + 6\left[-18 + 6\lambda - 8\right] + 2\left[24 - 14 + 2\lambda\right] = 0$$

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$\Rightarrow 8\lambda^2 - 80\lambda + 40 - \lambda^3 + 10\lambda^2 - 5\lambda + 36\lambda - 60 + 4\lambda + 20 = 0$

$\Rightarrow -(-\lambda^3 + 18\lambda^2 - 45\lambda = 0)$

$\Rightarrow \lambda^3 - 18\lambda^2 + 45\lambda = 0$

$\Rightarrow \lambda\{\lambda^2 - 18\lambda + 45) = 0$

45

3 15

$\Rightarrow \boxed{\lambda = 0}$

$\lambda^2 - 3\lambda - 15\lambda + 45 = 0$

$\lambda(\lambda - 3) - 15(\lambda - 3) = 0.$

$\boxed{\lambda = 3}$

$\boxed{\lambda = 15}$

Case(i):- $\lambda = 0$ Let $X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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8) Diagonalise the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

Sol) $|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$

$\Rightarrow (8-\lambda)[(7-\lambda)(3-\lambda) - 16] + 6[-6(3-\lambda) + 8]$
$+ 2[24 - 2(7-\lambda)] = 0.$

$\Rightarrow (8-\lambda)[21 - 7\lambda - 3\lambda + \lambda^2 - 16] + 6[-18 + 6\lambda + 8] +$
$2[24 - 14 + 2\lambda] = 0$

$\Rightarrow (8-\lambda)[\lambda^2 - 10\lambda + 5] + 6[6\lambda - 10] + 2[2\lambda + 10] = 0.$

$8\lambda^2 - 84\lambda + 40 - \lambda^3 + 10\lambda^2 - 5\lambda + 36\lambda - 40 + 4\lambda + 4\lambda = 0$

$= (-\lambda^3 + 18\lambda^2 - 45\lambda) = 0$

$\lambda^3 - 18\lambda^2 + 45\lambda = 0.$

$\lambda(\lambda^2 - 18\lambda + 45) = 0.$

$\boxed{\lambda = 0}$

$\lambda^2 - 3\lambda - 15\lambda + 45 = 0$

$\lambda(\lambda - 3) - 15(\lambda - 3) = 0.$

$\boxed{\lambda = 3}$ $\boxed{\lambda = 15}$

Case (i):- $\lambda = 0$ Let $X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$8x_1 - 6x_2 + 2x_3 = 0$ — (1)

$-6x_1 + 7x_2 - 4x_3 = 0$ — (2)

$2x_1 - 4x_2 + 3x_3 = 0$ — (3)

Taking (1) & (2).

$$\frac{x_1}{24 - 14} = \frac{-x_2}{-32 + 12} = \frac{x_3}{56 - 36}$$

$$\frac{x_1}{10} = \frac{+x_2}{+20} = \frac{x_3}{20}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

Case (ii):- $\lambda = 3$ Let $X_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{0+8} = \frac{-x_2}{+4} = \frac{x_3}{-20+12}$$

$$\frac{x_1}{8} = \frac{x_2}{4} = \frac{x_3}{-8}$$

$$X_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

Case (iii):—

$\lambda = 15$

$$X_3 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-7x_1 - 6x_2 + 2x_3 = 0$

$-6x_1 - 8x_2 - 4x_3 = 0$

$$\frac{x_1}{24+16} = \frac{-x_2}{28+12} = \frac{x_3}{56-36}$$

$$\frac{x_1}{\cancel{40}\,2} = \frac{-x_2}{\cancel{40}\,2} = \frac{x_3}{\cancel{20}\,1}$$

$$X_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

Modal Matrix:—

$$P = \begin{bmatrix} \overset{+}{1} & \overset{-}{2} & \overset{+}{2} \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$|P| = +1(1-4) - 2(2+4) + 2(-4-2)$

$= +1(-3) - 2(6) + 2(-6)$

$= -3 - 12 - 12$

$= -27 \neq 0$

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$$P^{-1} = \begin{bmatrix} 1/9 & 2/9 & 2/9 \\ 2/9 & 1/9 & -2/9 \\ 2/9 & -2/9 & 1/9 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

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*. Definations:-

1. Canonical form:- A Symmetric matrix is said to be in Canonical form if $Q = Y^T D Y$ where $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$.

Eg:-

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

$$[y_1 \quad y_2 \quad y_3] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$[0y_1 \quad 3y_2 \quad 15y_3] \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$0y_1^2 + 3y_2^2 + 15y_3^2$$

***

2. Orthogonal transformation:- The Vectors X_1, X_2, X_3 are said to be orthogonal to each other if

[i] $X_1^T X_2 = 0$ (or) $X_2^T X_1 = 0$

[ii] $X_2^T X_3 = 0$ (or) $X_3^T X_2 = 0$

[iii] $X_1^T X_3 = 0$ (or) $X_3^T X_1 = 0$.

3. **Normalised Eigen Vector:-**

Let $x = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$ be the Eigen vector then its normalised Eigen vector is denoted by $\|x\|$ [Norm] and it is given by $\|x\| = \begin{bmatrix} \frac{a_1}{\sqrt{a_1^2+b_1^2+c_1^2}} \\ \frac{b_1}{\sqrt{a_1^2+b_1^2+c_1^2}} \\ \frac{c_1}{\sqrt{a_1^2+b_1^2+c_1^2}} \end{bmatrix}$

Eg:- $x = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

$\|X\| = \begin{bmatrix} \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{bmatrix}$

4. **Rank (r):-** The no. of non-zero rows in diagonal matrix is the rank denoted by [r]

5. **Index(s):-** The no. of positive square terms in canonical form is the Index, denoted by [S]

6. **Signature:-** $2S-r$.

Eg:-

Rank $= r = 2$

Index $= S = 2$

Signature $= 2S-r$

$= 2(2) - 2$

$= 2$

7. Nature:-

[i] Positive definite:- If all the Eigen values are greater than zero then the symmetric matrix is said to be positive definite.

[ii] Negetive definite:- If all the Eigen values are less than zero then the symmetric matrix is said to be Negetive degree.

[iii] Positive Semi-definite:- A symmetric matrix is said to be positive semi-definite if atleast 1 Eigen value should be equal to zero.

[iv] Indefinite:- A symmetric matrix is said to be indefinite if eigen values are both [i.e +ve & -ve]

Q) Reduce the Quadratic form $q = 6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_1x_3$. into Canonical form, also find rank, index, & signature. & nature. by Orthogonal transform

Sol)

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$|A - \lambda I| = 0 \Rightarrow \begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix}$$

$\Rightarrow 6-\lambda\,[(3-\lambda)(3-\lambda)-1] + 2\,[-2(3-\lambda) + 1(2)] + 2\,(-2(-1) - 2(3-\lambda))$

$\Rightarrow 6-\lambda\,[9 - 3\lambda - 3\lambda + \lambda^2 - 1]$

$+2\,[-6 + 2\lambda + 2] + 2\,[2 - 6 + 2\lambda]$

$\Rightarrow 6-\lambda\,[\lambda^2 - 6\lambda + 8] + 2\,[2\lambda - 4] + 2\,[2\lambda - 4]$

$\Rightarrow [6\lambda^2 - 36\lambda + 48 - \lambda^3 + 6\lambda^2 - 8\lambda] + 4\lambda - 8 + 4\lambda - 8$

$\Rightarrow -\lambda^3 + 12\lambda^2 - 36\lambda + 32 = 0$

$\Rightarrow -[\lambda^3 - 12\lambda^2 + 36\lambda - 32]$

$\Rightarrow$ ~~$\lambda[\lambda^2 - 12\lambda + 36 - 32]$~~

$\Rightarrow -\lambda^3 + 12\lambda^2 - 36\lambda + 32 = 0.$

$-(\lambda^3 - 12\lambda^2 + 36\lambda - 32) = 0$.

Let $\lambda = 2$

$8 - 48 + 72 - 32 = 0$

$-80 + 80 = 0$

$$\begin{array}{c|cccc} 2 & 1 & -12 & 36 & -32 \\ & 0 & 2 & -20 & 32 \\ \hline & 1 & -10 & 16 & \boxed{0} \end{array}$$

$\lambda^2 - 10\lambda + 16 = 0$

$\lambda^2 - 8\lambda - 2\lambda + 16 = 0$

$\lambda(\lambda - 8) - 2(\lambda - 8) = 0$

$(\lambda - 8)(\lambda - 2)(\lambda - 2)$

$\lambda = 8, 2, 2.$

Case (i):- Let $\lambda = 8$; Let $X_1 = \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x1 - 2x2 + 2x3 = 0$

$2x1 - x2 - 5x3 = 0$

$$\frac{x1}{10+2} = \frac{-x2}{10-4} = \frac{x3}{2+4}$$

$$\frac{x1}{\cancel{12}} = \frac{x2}{-6} = \frac{x3}{\cancel{6}}$$

$$\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Case (ii):- $\lambda = 2$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x1 - x2 + x3 = 0$

Case (i):- $x_1 = 0$

$$-x_2 = -x_3$$

$$x_2 = x_3$$

$$\frac{x_1}{0} = \frac{x_2}{1} = \frac{x_3}{1}$$

$$X_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Case (ii):- $x_2 = 0$

$$2x_1 = -x_3$$

$$\frac{x_1}{-1} = \frac{x_3}{2}$$

$$\frac{x_1}{-1} = \frac{x_2}{0} = \frac{x_3}{2}$$

$$X_2 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \quad \|x_1\| = \begin{bmatrix} 2/\sqrt{6} \\ -1/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix} \quad \|x_2\| = \begin{bmatrix} -1/\sqrt{5} \\ 0 \\ 2/\sqrt{5} \end{bmatrix}$$

Let $X_3 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

$$X_2^T X_3 = \begin{bmatrix} -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = -1a + 0b + 2c = 0 \rightarrow (1)$$

$$X_1^T X_3 = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 2a - 1b + 1c = 0 \rightarrow (2)$$

$$\frac{a}{0+2} = \frac{-b}{-1-4} = \frac{c}{1}$$

$$\frac{a}{2} = \frac{b}{5} = \frac{c}{1} \qquad X_3 = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$$

$$\|x_3\| = \begin{bmatrix} 2/\sqrt{30} \\ 5/\sqrt{30} \\ 1/\sqrt{30} \end{bmatrix}$$

Modal Matrix (N):-

$$\begin{bmatrix} 2/\sqrt{6} & -1/\sqrt{5} & 1/\sqrt{6} \\ 1/\sqrt{5} & 0 & 1/\sqrt{6} \\ 2/\sqrt{30} & 5/\sqrt{30} & 1/\sqrt{30} \end{bmatrix}$$

$$N^T = \begin{bmatrix} 2/\sqrt{6} \\ -1/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$$

Modal Matrix (N):-
$$\begin{bmatrix} 2/\sqrt{6} & -1/\sqrt{5} & 2/\sqrt{30} \\ -1/\sqrt{6} & 0 & 5/\sqrt{30} \\ 1/\sqrt{6} & 2/\sqrt{5} & 1/\sqrt{30} \end{bmatrix}$$

$$N^T = \begin{bmatrix} 2/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} \\ -1/\sqrt{5} & 0 & 2/\sqrt{5} \\ 2/\sqrt{30} & 5/\sqrt{30} & 1/\sqrt{30} \end{bmatrix}$$

$$N^T A N = D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Canonical form:- Let $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$$Q = Y^T D Y = [y_1, y_2, y_3] = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$8y_1^2 + 2y_2^2 + 2y_3^2$$

Rank:- $r = 3$

Index:- $s = 3$

Signature:- $2s - r$
$= 6 - 3$
$= 3$

Nature:- Positive definite.

Q) $q = 3x^2+5y^2+3z^2-2xy-2yz+2xz = 0$

sol) $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ $\begin{matrix} x \\ y \end{matrix}$

$|A-\lambda I| = 0 \Rightarrow \begin{bmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{bmatrix}$

$\Rightarrow 3-\lambda\left[(5-\lambda)(3-\lambda)-1\right] +1\left[-1(3-\lambda)+1\right] +1\left[1-5+\lambda\right]$

$\Rightarrow 3-\lambda\left[15-5\lambda-3\lambda+\lambda^2-1\right] +1\left[-3+3\lambda+1\right] + 1-5+\lambda = 0$

$\Rightarrow 3-\lambda\left[\lambda^2-8\lambda+14\right] -3+3\lambda+1 \quad +1-5+\lambda = 0.$

$\Rightarrow 3\lambda^2-24\lambda+42-\lambda^3+8\lambda^2-14\lambda-3+3\lambda+2-5+\lambda = 0$

$\Rightarrow -\lambda^3+11\lambda^2-36\lambda+36 = 0.$

$\Rightarrow$ [illegible]

$\Rightarrow$ [illegible] $\qquad \lambda^3-11\lambda^2+36\lambda-36=0$

Let $\lambda = 2$

$\Rightarrow 8-44+72-36$

$80-80=0$

By Synthetic division.

	1	-11	36	-36
2	0	2	-18	36
	1	-9	18	0

$\lambda^2-9\lambda+18=0$

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$$\lambda^2 - 6\lambda - 3\lambda + 18 = 0$$

$$\lambda(\lambda - 6) - 3(\lambda - 6) = 0$$

$$(\lambda - 6) \quad (\lambda - 3) = 0$$

$$\lambda = 2, 6, 3.$$

Case 1:- If $\lambda = 2$ let $X1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 - x_2 + x_3 = 0$$
$$x_1 - x_2 + x_3 = 0$$

$$\frac{x_1}{-1+1} = \frac{-x_2}{1-1} = \frac{x_3}{-1+1}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 - x_2 + x_3 = 0$$
$$-x_1 + 3x_2 - x_3 = 0$$

$$\frac{x_1}{1-3} = \frac{-x_2}{-1+1} = \frac{x_3}{3-1}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\|x\| = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

Case 2:- If $\lambda = 3$ let $X1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} 0 & & \\ -1 & 2 & \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_1 + 2x_2 - x_3 = 0$$
$$x_1 - x_2 - 0 = 0$$

$$\frac{x_1}{0-1} = \frac{-x_2}{0+1} = \frac{x_3}{1-2}$$

$$\frac{x_1}{-1} = \frac{-x_2}{1} = \frac{x_3}{-1}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \quad \|x\| = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

Case 3:- $\lambda = 6$

$$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix}$$

$$-3x_1 - x_2 + x_3 = 0$$
$$-x_1 - x_2 - x_3 = 0$$

$$\frac{x_1}{1+1} = \frac{-x_2}{3+1} = \frac{x_3}{3-1}$$

$$\frac{x_1}{2} = \frac{x_2}{\cancel{4}} = \frac{x_3}{2}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad \|x_3\| = \begin{bmatrix} 1/\sqrt{6} \\ -2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$$

Modal Matrix $N = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & -2/\sqrt{6} \\ 1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix}$

$$N^T = \begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \end{bmatrix}$$

$$N^T A N = D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Conical form:- Let $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$$Q = y^T D y$$

$$[y_1 \ y_2 \ y_3] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$= 2y_1^2 + 3y_2^2 + 6y_3^2$

Rank $(r) = 3$

Index $(s) = 3$

Signature $= 2s - r = 2(3) - 3 = 3$

Nature = Positive definite.

Cayley - Hamilton Theorem

Statement :- "Every square matrix satisfies its characteristic equation"

Q) verify Cayley Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$$ also find its inverse

Sol: The char. Eqⁿ of A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & -1 & 0 \\ 3 & 1-\lambda & -1 \\ 2 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)\left[(1-\lambda)(3-\lambda)\right] + 1\left[3(3-\lambda)+2\right] = 0$$

$$\Rightarrow (2-\lambda)\left[3 - \lambda - 3\lambda + \lambda^2\right] + 1\left[9 - 3\lambda + 2\right] = 0$$

$$\Rightarrow (2-\lambda)\left[\lambda^2 - 4\lambda + 3\right] + 11 - 3\lambda = 0$$

$$\Rightarrow 2\lambda^2 - 8\lambda + 6 - \lambda^3 + 4\lambda^2 - 3\lambda + 11 - 3\lambda = 0$$

$$\Rightarrow -(-\lambda^3 + 6\lambda^2 - 14\lambda + 17) = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 14\lambda - 17 = 0 \quad \text{--- (1)}$$

$$A^2 = \begin{bmatrix} 1 & -3 & \cdots \\ 7 & -2 & \cdots \\ 10 & -2 & \cdots \end{bmatrix}$$

$$A^3 = \begin{bmatrix} -5 & -4 & \cdots \\ 0 & -9 & \cdots \\ 32 & -12 & \cdots \end{bmatrix}$$

If 'A' should satisfies (1) then CHT verified.

$$A^3 - 6A^2 + 14A - 17I = 0.$$

$$\begin{bmatrix} -5 & -4 & 6 \\ 0 & -9 & -10 \\ 32 & -12 & 29 \end{bmatrix} - \begin{bmatrix} 6 & -18 & 6 \\ 42 & -12 & -24 \\ 60 & -12 & 54 \end{bmatrix} + \begin{bmatrix} 28 & -14 & 0 \\ 42 & 14 & -14 \\ 28 & 0 & 42 \end{bmatrix} - \begin{bmatrix} 17 & 0 & 0 \\ 0 & 17 & 0 \\ 0 & 0 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

∴ CHT verified.

$A^3 - 6A^2 + 14A - 17I = 0.$

Multiply A^{-1} on B.S

$A^3A^{-1} - 6A^2A^{-1} + 14AA^{-1} - 17IA^{-1} = 0$

$A^2 - 6A + 14I - 17A^{-1} = 0.$

$A^2 - 6A + 14I = 17A^{-1}$

$$A^{-1} = \frac{1}{17}\left[\begin{matrix} 1 & -3 & 1 \\ 2 & -7 & -4 \\ 10 & -2 & 9 \end{matrix}\right] - \left[\begin{matrix} 12 & -6 & 0 \\ 18 & 6 & -6 \\ 12 & 0 & 18 \end{matrix}\right] + \left[\begin{matrix} 14 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 14 \end{matrix}\right]$$

$$A^{-1} = \frac{3}{17}$$

4) $A = \left[\begin{matrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{matrix}\right]$

Sol). The char. eq^n of A is $|A - \lambda I| = 0$

$\Rightarrow \left|\begin{matrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{matrix}\right| = 0$

$\Rightarrow (2-\lambda)[(2-\lambda)(2-\lambda) - 1] + 1[-1(2-\lambda) + 1] + 1(1 - 2 - \lambda) = 0$

$\Rightarrow (2-\lambda)[4 - 2\lambda - 2\lambda + \lambda^2 - 1] + 1[-2 + \lambda + 1] - 1 - \lambda = 0$

$\Rightarrow 2-\lambda[\lambda^2 - 4\lambda + 3] + \lambda - 2 - \lambda = 0$

$\Rightarrow 2\lambda^2 - 8\lambda + 6 - \lambda^3 + 4\lambda^2 - 3\lambda + \cancel{\lambda} - 2 - \cancel{\lambda} = 0$

$\Rightarrow -(-\lambda^3 + 6\lambda^2 - 11\lambda + 4 = 0)$

$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda + 4 = 0$

— (1)

$A^3 - 6A^2 + 9A - 4I = 0$

$$\begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - [\text{illegible}]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$A^3 - 6A^2 + 9A - 4I = 0.$

Multiply A^{-1} on B.S

$A^3A^{-1} - 6A^2A^{-1} + 9AA^{-1} - 4IA^{-1} = 0$

$A^2 - 6A + 9I - 4A^{-1} = 0$

$A^2 - 6A + 9I = 4A^{-1}$

$$A^{-1} = \frac{1}{4}\left[\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]$$

$$A^{-1} = \frac{1}{4}\left[\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}\right]$$

$$A^{-1} = \begin{bmatrix} 3/4 & 1/4 & -1/4 \\ 1/4 & 3/4 & 1/4 \\ -1/4 & 1/4 & 3/4 \end{bmatrix}$$

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