

Definitions:-

Matrix:- • The rectangular (or) square arrangement of real (or) complex no's is called Matrix.

• The square brackets $[]$ are used to represent the matrix.

• Eg:- $A = \begin{bmatrix} 2 & -i \\ 0 & 1 \end{bmatrix}_{2 \times 2}$, $A = \begin{bmatrix} 2i & -i \\ 0 & 1 \\ 2 & 3 \end{bmatrix}_{3 \times 2}$

Square Matrix:- • A matrix in which rows and columns are equal is called square matrix.

• Eg:- $A = \begin{bmatrix} 2 & -i \\ 0 & 1 \end{bmatrix}_{2 \times 2}$, $A = \begin{bmatrix} -1 & 1 & -1 \\ 2 & 3 & -1 \\ 3 & 1 & -4 \end{bmatrix}_{3 \times 3}$

Rectangular Matrix:- • A matrix in which rows and columns are not equal is called rectangular matrix.

• Eg:- $A = \begin{bmatrix} 2i & -i \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$

Identity Matrix:-

(or)

Unit Matrix

• A matrix in which the elements of diagonals are 1 & rest of the elements are 0 is called identity matrix. & it is denoted by I .

• Eg:- $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2 \times 2}$

Note:- Trace of a matrix.

⇒ The trace of a matrix is denoted by $\text{trace of } A$ and it is given by the sum of elements of diagonal.

⇒ Eg:-

$$A = \begin{bmatrix} -1 & 1 & -1 \\ 2 & 3 & -1 \\ 3 & 1 & -4 \end{bmatrix}$$
$$= -1 + 3 - 4$$
$$= -5 + 3 = -2$$

General representation of Matrix:-

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

*** Upper triangular matrix:-

⇒ A matrix which satisfies the condition $[a_{ij}] = 0$ when $i > j$ is called upper triangular matrix.
[or]

⇒ A matrix in which below diagonal elements are zero is called upper triangular matrix.

⇒ Eg:- Ex:-

$$\begin{bmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{bmatrix}$$

Lower triangular matrix:-

⇒ A matrix which satisfies the condition $[a_{ij}] = 0$ when $i < j$ is called lower triangular matrix.

⇒ Eg:- Ex:-

$$\begin{bmatrix} * & 0 & 0 \\ * & * & 0 \\ * & * & * \end{bmatrix}$$

Symmetric matrix:- A matrix which satisfies the following conditions is called Symmetric matrix.

[i] $a_{11} \neq a_{22} \neq a_{33}$

[ii] $a_{12} = a_{21}$
 $a_{32} = a_{23}$
 $a_{13} = a_{31}$

[or]
 $\Rightarrow$ A matrix which satisfies the condition $A^T = A$ then A is called Symmetric matrix.

eg:- $A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & -1 & -1 \\ 5 & -1 & 0 \end{bmatrix}$

Skew-Symmetric matrix:- A matrix which satisfies the following conditions is called skew symmetric matrix.

[i] $a_{11} = a_{22} = a_{33}$

[ii] $a_{12} + a_{21} = 0$

$a_{32} + a_{23} = 0$

$a_{13} + a_{31} = 0$

[or]
 $\Rightarrow$ A matrix which satisfies the condition $A^T = -A$ then A is called Skew Symmetric matrix.

eg:- $A = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 2 \\ -3 & -2 & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$

$-A = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$

$\therefore A^T = -A$

$\therefore A$ is skew symmetric matrix.

Rank

Determinants:-

$$1) A = \begin{bmatrix} + & - & + \\ 3 & 1 & 2 \\ 1 & 2 & -2 \\ 1 & 2 & 4 \end{bmatrix}$$

$$|A| = 3(8+4) - 1(4+2) + 2(2-2)$$

$$= 3(12) - 1(6)$$

$$= 36 - 6$$

$\Rightarrow$ Rank of A is 3

$$\rho(A) = 3$$

$$|A| = 30 \neq 0$$

$$2) A = \begin{bmatrix} + & - & + \\ 1 & 2 & 3 \\ 0 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix}$$

$$= 1(0+0) - 2(0-0) + 3(0-0)$$

$$|A| = 0$$

$\Rightarrow$ Rank of A is 1

$$\rho(A) = 1$$

$$3) A = \begin{bmatrix} + & - & + \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= 1(1-1) - 1(1-1) + 1(1-1)$$

$$|A| = 0$$

$\Rightarrow$ Rank of A is 1

$$\rho(A) = 1$$

$$4) A = \begin{bmatrix} + & - & + \\ 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

$$= 3(4-2) - 2(8-6) + 1(4-6)$$

$$= 3(2) - 2(2) + 1(-2)$$

$$= 6 - 4 - 2$$

$$= 0$$

$$A_1 = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= 3 - 4$$

$$= -1 \neq 0$$

$\Rightarrow$ Rank of A is 2

$$\rho(A) = 2$$

Rank of the matrix:-

$\Rightarrow$ The rank of the matrix is defined as the no. of non-zero rows in that matrix.

$\Rightarrow$ The rank of the matrix is denoted by $\rho(A)$.

Case 1:- If $|A| \neq 0$ then no. of non-zero rows is the rank.

Case 2:- If $|A| = 0$ and if there exist at least one submatrix whose determinant is not zero then rank of the matrix is equal to order of submatrix.

Case 3:- If $|A| = 0$ and determinant of every submatrix is also zero then rank is 1.

Rank of matrix:

$\Rightarrow$ Let 'A' represent a non-zero matrix, a number 'r' is said to be rank of the matrix A if:-

- 1) Every $(r+1)^{th}$ minor of A is zero.
- 2) Atleast 1 minor of order 'r' which is non-zero.

Echelon form:-

$\Rightarrow$ A $m \times n$ matrix is said to be in echelon form if:-

- 1) All zero rows (or) any zero row occurs below the non-zero row.
- 2) The no. of zeros before the first non-zero element in the row are in increasing order.

3) The first non-zero element in every row is unity [Optional].

19) Reduce the matrix into echelon form and find its rank.

(or)

Find the rank of the matrix.

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

Sol)

Given;

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

$$R_2 \rightarrow 3R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 2R_1$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{bmatrix}$$

$$R_3 \rightarrow \frac{R_3}{-2}$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$R_3 \rightarrow R_3 + R_2$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

No. of Non-zero rows = 2

$$\rho(A) = 2$$

$$\begin{bmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{bmatrix}$$

sol) Given;

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

$$= 3(4-2) - 2(8-6) + 1(4-6)$$

$$= 3(2) - 2(2) + 1(-2)$$

$$= 6 - 6$$

$$= 0$$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$$

$$= 4 - 2 = 2 \neq 0$$

$$\rho(A) = 2.$$

05-08-24.

29) $A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7 \end{bmatrix}$

sol) Given;

$$A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7 \end{bmatrix}$$

$$R_2 \rightarrow R_2 + 2R_1$$

$$R_3 \rightarrow R_3 + 3R_1$$

$$A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 0 & 5 & 1 & 16 \\ 0 & 8 & 4 & 31 \end{bmatrix}$$

$$R_3 \rightarrow 5R_3 - 8R_2$$

$$A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 0 & 5 & 1 & 16 \\ 0 & 0 & 12 & 27 \end{bmatrix}$$

$$R_3 \rightarrow \frac{R_3}{3}$$

$$A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 0 & 5 & 1 & 16 \\ 0 & 0 & 4 & 9 \end{bmatrix}$$

$\therefore$ The No. of non-zero rows
 $\rho(A) = 3$

Q3) Reduce the matrix into Echelon form

$$A = \begin{bmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{bmatrix}$$

Find the rank of matrix.

Sol)

$$|A| = -1(19-1) - 0(0) + 6(3+30)$$

$$= -1(17) + 6(33)$$

$$= -17 + 198 \neq 0$$

$$\rho(A) = 3$$

(or)

$$A = \begin{bmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{bmatrix}$$

$$R_2 \rightarrow R_2 + 3R_1$$

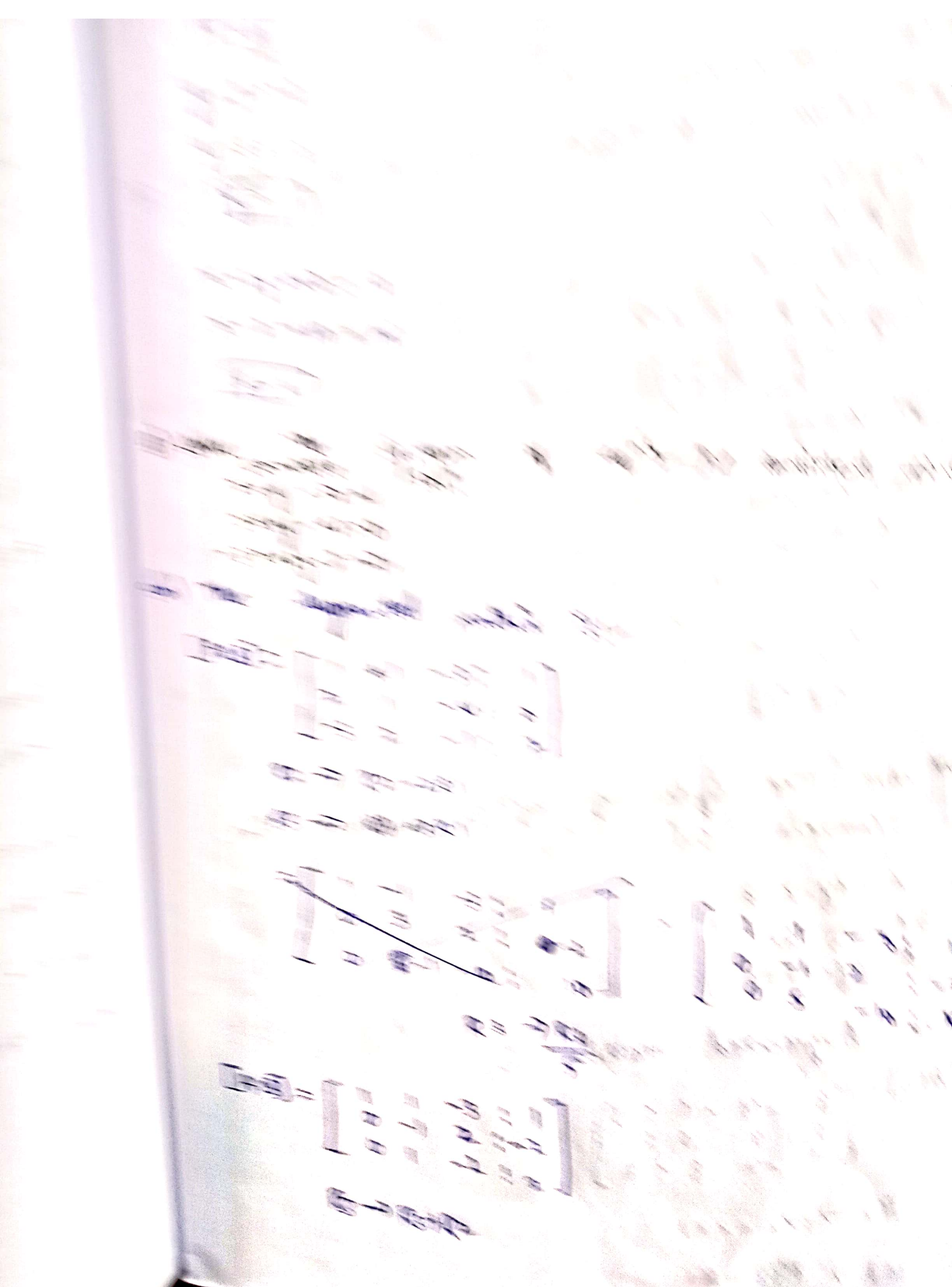
$$R_3 \rightarrow R_3 - 5R_1$$

$$A = \begin{bmatrix} -1 & 0 & 6 \\ 0 & 6 & 19 \\ 0 & 1 & -27 \end{bmatrix}$$

$$R_3 \rightarrow 6R_3 - R_2$$

$$A = \begin{bmatrix} -1 & 0 & 6 \\ 0 & 6 & 19 \\ 0 & 0 & -191 \end{bmatrix}$$

$\therefore$ No. of non-zero rows = 3
 $\rho(A) = 3$



09/01/24.

Solutions of linear System:-

Consider the system of Eq^{n's} are:-

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

①. The matrix form is

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$AX = B.$$

②. The Augmented matrix is

$$[A:B] = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix}$$

Case(i):- $\rho(A) = \rho(A:B) = n$

Then the system is consistent & has unique sol^{n's}

Case(ii):- $\rho(A) = \rho(A:B) < n$

Then the system is consistent & has many sol^{n's}

Case(iii):- $\rho(A) \neq \rho(A:B)$

Then the system is inconsistent & No solution

Q) solve the system of eqns:-

$$x - 2y + 4z = 12$$

$$2x - y + 5z = 18$$

$$-x + 3y - 3z = -8$$

Sol) The Augmented matrix is:-

$[A:B]$

$$= \begin{bmatrix} 1 & -2 & 4 & : & 12 \\ 2 & -1 & 5 & : & 18 \\ -1 & 3 & -3 & : & -8 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 + R_1$$

$$= \begin{bmatrix} 1 & -2 & 4 & : & 12 \\ 0 & 3 & -3 & : & -6 \\ 0 & 1 & 1 & : & 4 \end{bmatrix}$$

$$R_2 \rightarrow \frac{R_2}{3}$$

$$[A:B] = \begin{bmatrix} 1 & -2 & 4 & : & 12 \\ 0 & 1 & -1 & : & -2 \\ 0 & 1 & 1 & : & 4 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$[A:B] = \begin{bmatrix} 1 & -2 & 4 & : & 12 \\ 0 & 1 & -1 & : & -2 \\ 0 & 0 & 2 & : & 6 \end{bmatrix}$$

$$\rho(A) = 3$$

$$\rho(A:B) = 3$$

$$n = 3$$

$$\rho(A) = \rho(A:B) = n$$

$\therefore$ The system is consistent & it has unique solⁿs.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow (\sin)^2 = (\cos)^2$$

5. $\frac{1}{x^2} = x^{-2}$ $\frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$

$$1 = 28 - 17 + 25$$

5-5547

$$5 - 7 \text{ 5th } \checkmark$$

$$(x^2 - 5x + 6) \cdot (x^2 + 5x + 6) =$$

$$[x + 6 = 8]$$

123456789

1. $1000 - 200 = 800$

127

9/4/17

21.5/9m

... 5000 ...

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 229. 230. 231. 232. 233. 234. 235. 236. 237. 238. 239. 240. 241. 242. 243. 244. 245. 246. 247. 248. 249. 250. 251. 252. 253. 254. 255. 256. 257. 258. 259. 260. 261. 262. 263. 264. 265. 266. 267. 268. 269. 270. 271. 272. 273. 274. 275. 276. 277. 278. 279. 280. 281. 282. 283. 284. 285. 286. 287. 288. 289. 290. 291. 292. 293. 294. 295. 296. 297. 298. 299. 300. 301. 302. 303. 304. 305. 306. 307. 308. 309. 310. 311. 312. 313. 314. 315. 316. 317. 318. 319. 320. 321. 322. 323. 324. 325. 326. 327. 328. 329. 330. 331. 332. 333. 334. 335. 336. 337. 338. 339. 340. 341. 342. 343. 344. 345. 346. 347. 348. 349. 350. 351. 352. 353. 354. 355. 356. 357. 358. 359. 360. 361. 362. 363. 364. 365. 366. 367. 368. 369. 370. 371. 372. 373. 374. 375. 376. 377. 378. 379. 380. 381. 382. 383. 384. 385. 386. 387. 388. 389. 390. 391. 392. 393. 394. 395. 396. 397. 398. 399. 400. 401. 402. 403. 404. 405. 406. 407. 408. 409. 410. 411. 412. 413. 414. 415. 416. 417. 418. 419. 420. 421. 422. 423. 424. 425. 426. 427. 428. 429. 430. 431. 432. 433. 434. 435. 436. 437. 438. 439. 440. 441. 442. 443. 444. 445. 446. 447. 448. 449. 450. 451. 452. 453. 454. 455. 456. 457. 458. 459. 460. 461. 462. 463. 464. 465. 466. 467. 468. 469. 470. 471. 472. 473. 474. 475. 476. 477. 478. 479. 480. 481. 482. 483. 484. 485. 486. 487. 488. 489. 490. 491. 492. 493. 494. 495. 496. 497. 498. 499. 500. 501. 502. 503. 504. 505. 506. 507. 508. 509. 510. 511. 512. 513. 514. 515. 516. 517. 518. 519. 520. 521. 522. 523. 524. 525. 526. 527. 528. 529. 530. 531. 532. 533. 534. 535. 536. 537. 538. 539. 540. 541. 542. 543. 544. 545. 546. 547. 548. 549. 550. 551. 552. 553. 554. 555. 556. 557. 558. 559. 560. 561. 562. 563. 564. 565. 566. 567. 568. 569. 570. 571. 572. 573. 574. 575. 576. 577. 578. 579. 580. 581. 582. 583. 584. 585. 586. 587. 588. 589. 590. 591. 592. 593. 594. 595. 596. 597. 598. 599. 600. 601. 602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840. 84

-1-12

0 5

$$= \begin{bmatrix} 1 & 1 & -3 & : & 1 \\ 0 & -1 & 2 & : & -2 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$$\rho(A) = \rho(A:B) < n$$

∴ The system is consistent but it has infinite solⁿ

$$x + y - 3z = 1$$

$$-y + 2z = -2$$

$$-y + 2k = -2$$

$$-y = -2 - 2k$$

$$\boxed{y = 2 + 2k}$$

$$\text{Let } \boxed{z = k}$$

$$x + 2 + 2k - 3k = 1$$

$$x - k = 1 - 2$$

$$x - k = -1$$

$$x = -1 + k$$

$$\boxed{x = k - 1}$$

Q) Solve the system of eqⁿs. It is consistent find complete solⁿ

$$3x - 4y - z = 3$$

$$2x - 3y + z = 1$$

$$x - 2y + 3z = 2$$

Sol) The Augmented matrix is

$$[A:B] = \left[\begin{array}{ccc|c} 3 & -4 & -1 & 3 \\ 2 & -3 & 1 & 1 \\ 1 & -2 & 3 & 2 \end{array} \right]$$

$$R_2 \rightarrow 3R_2 - 2R_1$$

$$R_3 \rightarrow 3R_3 - R_1$$

$$= \begin{bmatrix} 3 & -4 & -1 & : & 3 \\ 0 & -1 & 5 & : & -3 \\ 0 & -2 & 10 & : & 3 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 2R_2$$

$$[A:B] = \begin{bmatrix} 3 & -4 & -1 & : & 3 \\ 0 & -1 & 5 & : & -3 \\ 0 & 0 & 0 & : & 9 \end{bmatrix}$$

$$\rho(A) = 2 \quad \rho(A:B) = 3$$

$$\rho(A) \neq \rho(A:B)$$

$\therefore$ The system is inconsistent & has NO solution.

9) Solve the system of eqns.

$$x + y + z + t = 4$$

$$x - z + 2t = 2$$

$$y + z - 3t = -1$$

$$x + 2y - z + t = 3$$

Sol) The Augmented matrix

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & 1 & : & 4 \\ 1 & 0 & -1 & 2 & : & 2 \\ 0 & 1 & 1 & -3 & : & -1 \\ 1 & 2 & -1 & 1 & : & 3 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_4 \rightarrow R_4 - R_1$$

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & 1 & : & 4 \\ 0 & -1 & -2 & 1 & : & -2 \\ 0 & 1 & 1 & -3 & : & -1 \\ 0 & 1 & -2 & 0 & : & -1 \end{bmatrix}$$

$$R_3 \rightarrow R_3 + R_2$$

$$R_4 \rightarrow R_4 + R_2$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & 1 & -2 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & -4 & 1 & -3 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - 4R_3$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & 1 & -2 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & 0 & -3 & 9 \end{bmatrix}$$

$$\rho(A) = 4$$

$$\rho(A:B) = 4$$

$$9 \quad t = 9 \Rightarrow \boxed{t=1}$$

$$-Z - 2t = -3$$

$$-Z - 2 = -3$$

$$-Z = -3 + 2$$

$$\boxed{Z = -1}$$

$$-y - 2Z + t = -2$$

$$-y - x + 1 = -2$$

$$\boxed{y = 1}$$

$$x + y + z + t = 4$$

$$\boxed{x = 1}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & 1 & -2 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & 0 & -3 & 9 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - 4R_3$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & 1 & -2 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & 0 & -3 & 9 \end{bmatrix}$$

$$= [A:B]$$

$$\rho(A:B) = 4$$

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$$\rho(A) = 4$$

$$\rho(A) = 4$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & 1 & -2 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & 0 & -3 & 9 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & 1 & -2 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & 0 & -3 & 9 \end{bmatrix}$$

$$\rho(A) = 4$$

$$\rho(A) = 4$$

$$g) 2x - y - z = 2$$

$$x + 2y + z = 2$$

$$4x - 7y - 5z = 2$$

So, The augmented matrix is

$$[A:B] = \left[\begin{array}{ccc|c} 2 & -1 & -1 & 2 \\ 1 & 2 & 1 & 2 \\ 4 & -7 & -5 & 2 \end{array} \right]$$

$$R_2 \rightarrow 2R_2 - R_1$$

$$R_3 \rightarrow R_3 - 2R_1$$

$$[A:B] = \left[\begin{array}{ccc|c} 2 & -1 & -1 & 2 \\ 0 & 5 & 3 & 2 \\ 0 & -5 & -3 & -2 \end{array} \right]$$

$$R_3 \rightarrow R_3 + R_2$$

$$[A:B] = \left[\begin{array}{ccc|c} 2 & -1 & -1 & 2 \\ 0 & 5 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\rho(A) = 2 ; \rho(A:B) = 2$$

$$\rho(A) = \rho(A:B) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

$$2x - y - z = 2$$

$$5y + 3z = 2$$

$$5y + 3K = 2$$

$$5y = 2 - 3K$$

$$\boxed{y = \frac{2-3K}{5}}$$

$$2x - \left(\frac{2-3K}{5}\right) - K = 2$$

$$10x - 2 + 3K - 5K = 10$$

$$10x - 8K = 12$$

$$10x - 2 + 3K - 5K = 10$$

H/W:-

$$2(2) + 1$$

$$2(1) + 1$$

$$2(2) - 2$$

$$-7 - 2(-1)$$

$$-7 + 2$$

$$-5 - 2(-1)$$

$$-5 + 2$$

$$2 - 2(2)$$

$$x + 0 = x$$

$$\boxed{x + 0 = x}$$

$$\begin{aligned} d &= 7 + 4K \\ d &= 5 + 3K \\ d &= 5 + 3K \end{aligned}$$

$$\rho(A) = \rho(A:B)$$

$$\rho(A) = \rho(A:B)$$

$$\rho(A) = \rho(A:B)$$

$$\rho(A) = \rho(A:B)$$

$$\boxed{\text{Let } z = K}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & -1 & 2 \\ 0 & 5 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$18 - 5K = 5K$$

$$18 - 5K = 5K$$

$$\left[\begin{array}{ccc|c} 2 & -1 & -1 & 2 \\ 0 & 5 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$18 - 5K = 5K$$

$$10x - 2K = 10 + 2$$

$$10x - 2K = 12$$

$$10x = 12 + 2K$$

$$\frac{5}{10}x = \frac{12 + 2K}{10}$$

$$5x = 6 + K$$

$$x = \frac{6 + K}{5}$$

10-09-24.

Q) Find the values of a & b for which the system of eqⁿ is

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + az &= b \end{aligned}$$

- (i) Unique solⁿ.
- (ii) Infinite solⁿ.
- (iii) No solⁿ.

Sol) The matrix form is $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ b \end{bmatrix}$

Augmented matrix.

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & a & b \end{array} \right]$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & a-1 & b-6 \end{array} \right]$$

$$R_3 \rightarrow R_3 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & a-3 & b-10 \end{array} \right]$$

$$1x + 1y + 1z = 6$$

$$1x + 1y + 1z = 6$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x + y + z = 6$$

$$x + y + z = 6$$

$$x + y + z = 6$$

$$x + y + z = 6$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x + y + z = 6$$

$$x + y + z = 6$$

$$x + y + z = 6$$

$$x + y + z = 6$$

$$= \begin{bmatrix} -1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & a-3 & : & b-10 \end{bmatrix}$$

Case(i):- $a=3$; $b=10$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$$\rho(A)=2; \rho(A:B)=2$$

$$\rho(A) = \rho(A:B) < n$$

Infinite solⁿ.

Case(ii):- $a \neq 3$; $b=10$
($a=2$)

$$= \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & -1 & : & 0 \end{bmatrix}$$

$$\rho(A)=3; \rho(A:B)=3$$

$$\rho(A) = \rho(A:B) = n$$

Unique Solⁿ.

Case(iii):- $a=3$; $b \neq 10$
($b=9$)

$$= \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & 0 & : & -1 \end{bmatrix}$$

$$\rho(A)=2; \rho(A:B)=3$$

$$\rho(A) \neq \rho(A:B)$$

No solⁿ.

$$\begin{bmatrix} 0 & 1 & 2 & : & 4 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 2 & : & 4 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 2 & : & 4 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 2 & : & 4 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$$\rho(A)=2; \rho(A:B)=3$$

$$\rho(A) \neq \rho(A:B)$$

$$9) \quad x - 3y - 8z + 10 = 0$$

$$3x + y - 4z = 0$$

$$2x + 5y + 6z - 13 = 0.$$

Q.1)

Augmented matrix

$$[A:B] = \begin{bmatrix} 1 & -3 & -8 & : & -10 \\ 3 & 1 & -4 & : & 0 \\ 2 & 5 & 6 & : & 13 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 3R_1$$

$$R_3 \rightarrow R_3 - 2R_1$$

$$[A:B] = \begin{bmatrix} 1 & -3 & -8 & : & -10 \\ 0 & 10 & 20 & : & 30 \\ 0 & 11 & 22 & : & 33 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$R_3 \rightarrow \frac{R_3}{11} ; R_2 \rightarrow \frac{R_2}{10}$$

$$[A:B] = \begin{bmatrix} 1 & -3 & -8 & : & -10 \\ 0 & 1 & 2 & : & 3 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$$[A:B] = \begin{bmatrix} 1 & -3 & -8 & : & -10 \\ 0 & 1 & 2 & : & 3 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$$\rho(A) = 2 ; \rho(A:B) = 2$$

$$\rho(A) = \rho(A:B) < n$$

Infinite solⁿ

$$\Rightarrow x - 3y - 8z = -10$$

$$3x + y - 4z = 0$$

$$2x + 5y + 6z = 13$$

$$r = \rho(A) ; s = \rho(A)$$

$$r > s \Rightarrow \rho(A) > \rho(A)$$

Not infinite

$$r = s ; r \neq n \Rightarrow \rho(A) = \rho(A) < n$$

$$\begin{bmatrix} 1 & 0 & 0 & : & 0 \\ 0 & 1 & 0 & : & 0 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$$r = \rho(A) ; s = \rho(A)$$

$$r = \rho(A) ; s = \rho(A)$$

Not infinite

$$r \neq s ; r = n \Rightarrow \rho(A) \neq \rho(A) = n$$

$$\begin{bmatrix} 1 & 0 & 0 & : & 0 \\ 0 & 1 & 0 & : & 0 \\ 0 & 0 & 1 & : & 0 \end{bmatrix}$$

$$r = \rho(A) ; s = \rho(A)$$

$$r = \rho(A) ; s = \rho(A)$$

Not infinite

$$9) x + y + z = 6$$

$$2x + 3y - 2z = 2$$

$$5x + y + 2z = 13$$

501) The matrix form:-

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -2 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 13 \end{bmatrix}$$

Augmented matrix

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 2 & 3 & -2 & : & 2 \\ 5 & 1 & 2 & : & 13 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 5R_1$$

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & -4 & : & -10 \\ 0 & -4 & -3 & : & -17 \end{bmatrix}$$

$$R_3 \rightarrow R_3 + 4R_2$$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & -4 & : & -10 \\ 0 & 0 & -19 & : & -57 \end{bmatrix}$$

$$\rho(A) = 3; \rho(A:B) = 3$$

$$\rho(A) = \rho(A:B) = 3$$

Unique solⁿ.

20249.

Q) Find all the values of 'k' such that the rank of matrix

$$A = \begin{bmatrix} k & -1 & 0 & 0 \\ 0 & k & -1 & 0 \\ 0 & 0 & k & -1 \\ -6 & 0 & -6 & 1 \end{bmatrix}$$

is equal to '3'

Q1)

Given

$$\rho(A) = 3$$

$$|A| = 0$$

$$\begin{vmatrix} k & -1 & 0 & 0 \\ 0 & k & -1 & 0 \\ 0 & 0 & k & -1 \\ -6 & 11 & -6 & 1 \end{vmatrix} = 0$$

$$k \begin{vmatrix} k & -1 & 0 \\ 0 & k & -1 \\ 11 & -6 & 1 \end{vmatrix} + 1 \begin{vmatrix} 0 & -1 & 0 \\ 0 & k & -1 \\ -6 & -6 & 1 \end{vmatrix} + 0 \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} + 0 \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$$k [k(k-6) + 1(0+11)] + 1 [0(0) + 1(0-6)] = 0$$

$$k [k^2 - 6k + 11] - 6 = 0$$

$$k^3 - 6k^2 + 11k - 6 = 0$$

Let $K=1$

$$\begin{vmatrix} 1 & -6 & 11 & -6 \\ 0 & 1 & -5 & 6 \\ 0 & -5 & 6 & 0 \end{vmatrix}$$

$$k^2 - 5k + 6 = 0$$

$$k^2 - 2k - 3k + 6 = 0$$

$$k(k-2) - 3(k-2) = 0$$

$K=1$

$K=3$

$K=2$

9) Find the rank of matrix

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 4 & 6 \\ 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 1 & 1 & 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 3 & 4 \\ 6 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 3 & 2 \\ -4 & 0 & 3 & 5 \\ 7 & 2 & 1 & 1 \end{bmatrix}$$

561)

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 3R_1$$

$$R_4 \rightarrow R_4 - 6R_1$$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - R_2$$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3$$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 3.$$

2) $A = \begin{bmatrix} 3 & 1 & 4 & 6 \\ 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 1 & 1 & 2 & 2 \end{bmatrix}$

501)

$$R_2 \rightarrow 3R_2 - 2R_1$$

$$R_3 \rightarrow 3R_3 - 4R_1$$

$$3R_4 \rightarrow 3R_4 - R_1$$

$$A = \begin{bmatrix} 3 & 1 & 4 & 6 \\ 0 & 1 & -2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 2 & 2 & 4 \end{bmatrix}$$

$$R_1 \rightarrow R_1 - 3R_2$$

$$\begin{bmatrix} 0 & 0 & 10 & 18 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 \leftrightarrow R_3$$

$$R_2 \rightarrow R_2 - R_3$$

$$R_1 \rightarrow R_1 - 10R_3$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 \rightarrow R_1 - R_2$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 3$$

$$3 - 2$$

$$6 - 8$$

$$R_4 \rightarrow R_4 - R_3$$

$$\Rightarrow \begin{bmatrix} 3 & 1 & 4 & 6 \\ 0 & 1 & -2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 2R_2$$

$$-1 - 2(-2)$$

$$-1 + 4 = 3$$

$$\Rightarrow \begin{bmatrix} 3 & 1 & 4 & 6 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - R_3$$

$$\Rightarrow \begin{bmatrix} 3 & 1 & 4 & 6 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 3.$$

$$\begin{bmatrix} 0 & 8 & 5 & 1 \\ 1 & 2 & 4 & 5 \\ 2 & 1 & 3 & 6 \\ 3 & 1 & 8 & 2 \end{bmatrix}$$

$$R_2 \leftrightarrow R_1 \rightarrow R_1$$

$$R_3 - 2R_1 \rightarrow R_3$$

$$R_4 - 3R_1 \rightarrow R_4$$

$$\begin{bmatrix} 0 & 8 & 5 & 1 \\ 1 & 2 & 4 & 5 \\ 2 & 1 & 3 & 6 \\ 3 & 1 & 8 & 2 \end{bmatrix}$$

$$R_2 - R_1 \rightarrow R_2$$

$$\begin{bmatrix} 0 & 8 & 5 & 1 \\ 1 & 2 & 4 & 5 \\ 2 & 1 & 3 & 6 \\ 3 & 1 & 8 & 2 \end{bmatrix}$$

$$R_3 - 2R_2 \rightarrow R_3$$

$$\begin{bmatrix} 0 & 8 & 5 & 1 \\ 1 & 2 & 4 & 5 \\ 2 & 1 & 3 & 6 \\ 3 & 1 & 8 & 2 \end{bmatrix}$$

$$\rho(A) = 3$$

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = A$$

$$R_2 - R_1 \rightarrow R_2$$

$$R_3 - R_1 \rightarrow R_3$$

$$R_4 - R_1 \rightarrow R_4$$

11/09/24.

Q) If $A = \begin{bmatrix} 3 & x & 6 \\ 5 & 2 & x_1 \\ 6 & 4 & 8 \end{bmatrix}$ is Symmetric. Find x .

Sol) Given 'A' is Symmetric

$$\Rightarrow A^T = A$$

$$= \begin{bmatrix} 3 & 5 & 6 \\ x & 2 & 4 \\ 6 & 4 & 8 \end{bmatrix} = \begin{bmatrix} 3 & x & 6 \\ 5 & 2 & 4 \\ 6 & 4 & 8 \end{bmatrix}$$

$$\boxed{x = 5}$$

Q) If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ is Skew-symmetric. Find x .

Sol) Given 'A' is Skew-symmetric.

$$\Rightarrow A^T = -A$$

$$= \begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & x \\ 1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -1 \\ -2 & 0 & -2 \\ -1 & -x & 0 \end{bmatrix}$$

$$\boxed{x = 2}$$

Q) Find rank of $\begin{bmatrix} 4 & -2 & 2 \\ 5 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$

$$\text{Sol) Let } A = \begin{bmatrix} 4 & -2 & 2 \\ 5 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$$

$$|A| = +4(3-4) + 2(5-2) + 2(20-6)$$

$$= 4(-1) + 2(3) + 2(14)$$

$$= -4 + 6 + 28$$

$$|A| = 30 \neq 0$$

$$\rho(A) = 3$$

By Using Eclon form:-

501)

$$A = \begin{bmatrix} 4 & -2 & 2 \\ 5 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$$

$$R_1 \rightarrow \frac{R_1}{2}$$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 5 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$$

$$R_2 \rightarrow 2R_2 - 5R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 11 & -3 \\ 0 & 5 & 0 \end{bmatrix}$$

$$R_3 \rightarrow 11R_3 - 5R_2$$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 11 & -3 \\ 0 & 0 & 15 \end{bmatrix}$$

$$f(A) = 3$$

| | | |
|---|----|----|
| 5 | 5 | 10 |
| 1 | 2 | 2 |
| 1 | 10 | 5 |

19/09/24 * Eigen Values and Eigen Vectors:-

Eigen Values:- The roots of Characteristic eqⁿ $|A - \lambda I| = 0$ is called Eigen Values. / Latent roots / Characteristic roots.

* Non-Repeated Eigen values:-

g) Find the Eigen Values and corresponding Eigen vectors for the matrix

Sol) $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

The characteristic eqⁿ is

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda) [(2-\lambda)(5-\lambda)] = 0$$

$$\lambda = 2, 3, 5$$

Case (i):- $\lambda = 2$ Let $X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 6 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 + 4x_3 = 0 \quad \text{--- (1)}$$

$$0x_1 + 0x_2 + 6x_3 = 0 \quad \text{--- (2)}$$

$$0x_1 + 0x_2 + 3x_3 = 0 \quad \text{--- (3)}$$

Solving (1) & (2)

$$x_1 + x_2 + 4x_3 = 0$$

$$0x_1 + 0x_2 + 6x_3 = 0$$

$$x_1 + x_2 + 4x_3 = 0$$

$$0x_1 + 0x_2 + 6x_3 = 0$$

$$\frac{x_1}{1} = \frac{-x_2}{0} = \frac{-x_3}{6}$$

$$\frac{x_1}{1} = \frac{-x_2}{0} = \frac{-x_3}{6}$$

$$\frac{x_1}{1} = \frac{-x_2}{0} = \frac{-x_3}{6}$$

$$\frac{x_1}{6} = \frac{-x_2}{6} = \frac{-x_3}{6}$$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix} = A$$

$$x_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

Case (ii):

$$\lambda = 3$$

$$\text{Let } x_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 4 \\ 0 & -1 & 6 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0x_1 + x_2 + 4x_3 = 0$$

$$0x_1 - x_2 + 6x_3 = 0$$

$$\frac{x_1}{6+4} = \frac{-x_2}{0-0} = \frac{-x_3}{0-6}$$

$$\frac{x_1}{10} = \frac{x_2}{0} = \frac{x_3}{0}$$

$$x_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Case (iii):- Let $\lambda = 5$ Let $X_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} -2 & 1 & 4 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + x_2 + 4x_3 = 0$$

$$x_1 - 3x_2 + 6x_3 = 0$$

$$\frac{x_1}{6+12} = \frac{x_2}{+12} = \frac{x_3}{6}$$

$$\frac{x_1}{18} = \frac{x_2}{12} = \frac{x_3}{6}$$

$$X_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

9)

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Q1) The Char. eqⁿ is

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (8-\lambda) [(7-\lambda)(3-\lambda) - 16] + 6 [-6(3-\lambda) - 8] + 2 [-24 - 2(7-\lambda)] = 0$$

$$\Rightarrow (8-\lambda) [21 - 7\lambda - 3\lambda + \lambda^2 - 16] + 6 [-18 + 6\lambda - 8] + 2 [-24 - 14 + 2\lambda] = 0$$

$$\Rightarrow 8\lambda^2 - 80\lambda + 40 - 1^3 + 10\lambda^2 - 5\lambda + 36\lambda - 66 + 4\lambda + 20 = 0$$

$$\Rightarrow -(-\lambda^3 + 18\lambda^2 - 45\lambda = 0)$$

$$\Rightarrow \lambda^3 - 18\lambda^2 + 45\lambda = 0$$

$$\Rightarrow \lambda (\lambda^2 - 18\lambda + 45) = 0$$

$$\Rightarrow \boxed{\lambda = 0}$$

$$\lambda^2 - 3\lambda - 15\lambda + 45 = 0$$

$$\lambda(\lambda - 3) - 15(\lambda - 3) = 0$$

$$\boxed{\lambda = 3}$$

$$\boxed{\lambda = 15}$$

Case (i): $\lambda = 0$

Let $X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

19/9/24

9) Diagonalise the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

sol)

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (8-\lambda) [(7-\lambda)(3-\lambda) - 12] + 6 [-6(3-\lambda) + 8] + 2 [24 - 2(7-\lambda)] = 0$$

$$\Rightarrow (8-\lambda) [\lambda^2 - 10\lambda + 5] + 6 [-18 + 6\lambda + 8] + 2 [24 - 14 + 2\lambda] = 0$$

$$\Rightarrow (8-\lambda) [\lambda^2 - 10\lambda + 5] + 6 [6\lambda - 10] + 2 [2\lambda + 10] = 0$$

$$8\lambda^3 - 25\lambda + 40 = 0 \Rightarrow \lambda^3 + 18\lambda^2 = 5\lambda + 36 \Rightarrow \lambda = 40 \Rightarrow 40\lambda + 40 = 0$$

$$= (-\lambda^3 + 18\lambda^2 - 40\lambda) = 0$$

$$\lambda^3 - 18\lambda^2 + 40\lambda = 0$$

$$\lambda(\lambda^2 - 18\lambda + 40) = 0$$

$$\boxed{\lambda = 0}$$

$$\lambda^3 - 3\lambda - 15\lambda + 40 = 0$$

$$\lambda(\lambda - 3) - 15(\lambda - 3) = 0$$

$$\boxed{\lambda = 3}$$

$$\boxed{\lambda = 15}$$

(0, 0, 1):

$$\lambda = 0$$

$$\text{Let } X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 8-0 & 2 \\ -6 & 3-0 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$8x_1 - 6x_2 + 2x_3 = 0 \quad \text{--- (1)}$$

$$-6x_1 + 3x_2 - 4x_3 = 0 \quad \text{--- (2)}$$

$$2x_1 - 4x_2 + 3x_3 = 0 \quad \text{--- (3)}$$

Taking (1) & (2),

$$\frac{x_1}{24-14} = \frac{-x_2}{-32+12} = \frac{x_3}{56-26}$$

$$\frac{x_1}{10} = \frac{x_2}{-20} = \frac{x_3}{30}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

(0, 0, 1):

$$\lambda = 3$$

$$\text{Let } X_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{0+8} = \frac{-x_2}{+4} = \frac{x_3}{-20+12}$$

$$\frac{x_1}{8} = \frac{x_2}{-4} = \frac{x_3}{-8}$$

$$X_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

Case (iii): -

$$\lambda = 15$$

$$X_3 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} -7x_1 - 6x_2 + 2x_3 &= 0 \\ -6x_1 - 8x_2 - 4x_3 &= 0 \end{aligned}$$

$$\frac{x_1}{24+16} = \frac{-x_2}{28+12}$$

$$= \frac{x_3}{56-36}$$

$$\frac{x_1}{40} = \frac{-x_2}{40} = \frac{x_3}{20}$$

$$X_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

Model matrix:-

$$P = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$|P| = +1(1-4) - 2(2+4) + 2(-4-2)$$

$$= +1(-3) - 2(6) + 2(-6)$$

$$= -3 - 12 - 12$$

$$= -27 \neq 0$$

$$P^{-1} = \begin{bmatrix} 1/9 & 2/9 & 2/9 \\ 2/9 & 1/9 & -2/9 \\ 2/9 & -2/9 & 1/9 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

23/09/24

* Definitions:-

1. Canonical form:-

A symmetric matrix is said to be in Canonical form

$$Q = Y^T D Y$$

Eg:-

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

$$\begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 0y_1 & 3y_2 & 15y_3 \end{bmatrix}$$

$$0y_1^2 + 3y_2^2 + 15y_3^2$$

2. Orthogonal transformation:-

The vectors x_1, x_2, x_3 are said to be orthogonal to each other if

(i) $x_1^T x_2 = 0$

(or) $x_2^T x_1 = 0$

(ii) $x_2^T x_3 = 0$

(or) $x_3^T x_2 = 0$

(iii) $x_1^T x_3 = 0$

(or) $x_3^T x_1 = 0$

3. Normalised Eigen Vectors:-

Let $x = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$ be the eigen vector then its normalised Eigen vector is denoted by $\|x\|$ and it is given by $\|x\| = \frac{1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}$

$$\|x\| = \begin{bmatrix} \frac{a_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} \\ \frac{b_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} \\ \frac{c_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} \end{bmatrix}$$

Eg:- $x = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

$$\|x\| = \begin{bmatrix} \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{bmatrix}$$

4. Rank (r):- The no. of non-zero rows in diagonal matrix is the rank denoted by $[r]$

5. Index (s):- The no. of positive square terms in canonical form is the Index denoted by $[s]$

6. Signature:- $2s - r$

Eg:-

$$\text{Rank} = r = 2$$

$$\text{Index} = s = 2$$

$$\text{Signature} = 2s - r$$

$$= 2(2) - 2$$

$$= 2$$

7. Nature:-

- (i) Positive definite:- If all the Eigen values are greater than zero then the symmetric matrix is said to be positive definite.
- (ii) Negative definite:- If all the Eigen values are less than zero then the symmetric matrix is said to be Negative definite.
- (iii) Positive Semi-definite:- A symmetric matrix is said to be positive semi-definite if at least 1 Eigen value should be equal to zero.
- (iv) Indefinite:- A symmetric matrix is said to be indefinite if [Eigen values are both [i.e. +ve & -ve]

Q) Reduce the Quadratic form $q = 6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_1x_3$. into Canonical form, also find rank, index, Signature. & nature. by orthogonal transform

Sol) $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix}$$

$$\Rightarrow 6-\lambda [(3-\lambda)(3-\lambda)] + 2 [-2(3-\lambda) + 1(2)] + 2 (-2(-1) - 2(3-\lambda))$$

$$\Rightarrow 6-\lambda [9 - 3\lambda - 3\lambda + \lambda^2] + 2 [-6 + 2\lambda + 2] + 2 [2 - 6 + 2\lambda]$$

$$\Rightarrow 6-\lambda [\lambda^2 - 6\lambda + 9] + 2 [2\lambda - 4] + 2 [2\lambda - 4]$$

$$\Rightarrow [6\lambda^2 - 36\lambda + 48 - \lambda^3 + 6\lambda^2 - 8\lambda] + 4\lambda - 8 + 4\lambda - 8$$

$$\Rightarrow -\lambda^3 + 12\lambda^2 - 36\lambda + 32 = 0$$

$$\Rightarrow -[\lambda^3 - 12\lambda^2 + 36\lambda - 32]$$

$$\Rightarrow \lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$$

$$\Rightarrow -\lambda^3 + 12\lambda^2 - 36\lambda + 32 = 0$$

$$-(\lambda^3 - 12\lambda^2 + 36\lambda - 32) = 0$$

Let $\lambda = 2$

$$8 - 48 + 72 - 32 = 0$$

$$-80 + 80 = 0$$

$$\begin{array}{c|ccc} 2 & 1 & -12 & 36 & -32 \\ & 0 & 2 & -20 & 32 \\ \hline & 1 & -16 & 16 & 0 \end{array}$$

$$\lambda^2 - 16\lambda + 16 = 0$$

$$\lambda^2 - 8\lambda - 2\lambda + 16 = 0$$

$$\lambda(\lambda - 8) - 2(\lambda - 8) = 0$$

$$(\lambda - 8)(\lambda - 2)(\lambda - 2)$$

$$\lambda = 8, 2, 2$$

Case (i):- Let $\lambda = 8$; - Let $x_i = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 - 2x_2 + 2x_3 = 0$$

$$-2x_1 - x_2 - 5x_3 = 0$$

$$\frac{x_1}{10+2} = \frac{-x_2}{10-4} = \frac{x_3}{2+4}$$

$$\frac{x_1}{12} = \frac{-x_2}{6} = \frac{x_3}{6}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Case (ii):- $\lambda = 2$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 - x_2 + x_3 = 0$$

Case (i): $x_1 = 0$

$$-x_2 = -x_3$$

$$x_2 = x_3$$

$$\frac{x_1}{0} = \frac{x_2}{1} = \frac{x_3}{1}$$

$$X_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$\|X_1\| = \begin{bmatrix} 2/\sqrt{6} \\ -1/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$$

$$\|X_2\| = \begin{bmatrix} -1/\sqrt{5} \\ 0 \\ 2/\sqrt{5} \end{bmatrix}$$

Let $X_3 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$$X_2^T X_3 = [-1 \ 0 \ 2] \begin{bmatrix} a \\ b \\ c \end{bmatrix} = -1a + 0b + 2c = 0 \rightarrow (1)$$

$$X_1^T X_3 = [2 \ -1 \ 1] \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 2a - b + c = 0 \rightarrow (2)$$

$$\|X_3\| = \begin{bmatrix} 2/\sqrt{30} \\ 5/\sqrt{30} \\ 1/\sqrt{30} \end{bmatrix}$$

$$\frac{a}{0+2} = \frac{-b}{-1+4} = \frac{c}{1} = \frac{1}{1}$$

$$\frac{a}{2} = \frac{b}{3} = \frac{c}{1} = 1$$

$$X_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

Modal Matrix (N):

$$N^T = \begin{bmatrix} 2/\sqrt{6} & -1/\sqrt{5} & 1/\sqrt{30} \\ -1/\sqrt{6} & 0 & 5/\sqrt{30} \\ 1/\sqrt{6} & 2/\sqrt{5} & 1/\sqrt{30} \end{bmatrix}$$

Modal Matrix (N):-

$$N = \begin{bmatrix} 2/\sqrt{6} & -1/\sqrt{5} & 2/\sqrt{30} \\ -1/\sqrt{6} & 0 & 5/\sqrt{30} \\ 1/\sqrt{6} & 2/\sqrt{5} & 1/\sqrt{30} \end{bmatrix}$$

$$N^T = \begin{bmatrix} 2/\sqrt{6} & -1/\sqrt{5} & 1/\sqrt{6} \\ -1/\sqrt{6} & 0 & 2/\sqrt{5} \\ 2/\sqrt{30} & 5/\sqrt{30} & 1/\sqrt{30} \end{bmatrix}$$

$$N^T A N = D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Canonical form:-

$$\text{Let } Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$Q = Y^T P Y$$

$$= [y_1, y_2, y_3] \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$8y_1^2 + 2y_2^2 + 2y_3^2$$

Rank:- $n = 3$

Index:- $S = 3$

Signature:- $2S - n$
 $= 6 - 3$
 $= 3$

Nature:- Positive definite.

$$9) \quad q = 3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2xz = 0$$

$$10) \quad A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$$(A - \lambda I) = 0 \Rightarrow \begin{bmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{bmatrix}$$

$$\Rightarrow 3-\lambda \left[(5-\lambda)(3-\lambda)-1 \right] - 1 \left[-1(3-\lambda)+1 \right] + 1 \left[1-5+\lambda \right]$$

$$\Rightarrow 3-\lambda \left[15-5\lambda-3\lambda+\lambda^2-1 \right] - 1 \left[-3+3\lambda+1 \right] + 1-5+\lambda = 0$$

$$\Rightarrow 3-\lambda \left[\lambda^2 - 8\lambda + 14 \right] - 3 + 3\lambda + 1 + 1 - 5 + \lambda = 0$$

$$\Rightarrow 3\lambda^2 - 24\lambda + 42 - \lambda^3 + 8\lambda^2 - 14\lambda - 3 + 3\lambda + 1 - 5 + \lambda = 0$$

$$\Rightarrow -\lambda^3 + 11\lambda^2 - 36\lambda + 36 = 0$$

$$\Rightarrow \lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0$$

$$\Rightarrow \lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0$$

$$\Rightarrow 8 - 44 + 72 - 36 = 0$$

$$80 - 80 = 0$$

By Synthetic division.

| | | | | |
|---|---|-----|-----|-----|
| | 1 | -11 | 36 | -36 |
| 2 | 0 | 2 | -18 | 36 |
| | 1 | -9 | 18 | 0 |

$$\lambda^2 - 9\lambda + 18 = 0$$

$$\lambda^2 - 6\lambda - 3\lambda + 18 = 0$$

$$\lambda(\lambda - 6) - 3(\lambda - 6) = 0$$

$$(\lambda - 6)(\lambda - 3) = 0$$

$$\lambda = 2, 6, 3.$$

Case 1:-

$$\text{If } \lambda = 2$$

$$\text{let } x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 - x_2 + x_3 = 0$$

$$x_1 - x_2 + x_3 = 0$$

$$x_1 - x_2 + x_3 = 0$$

$$-x_1 + 3x_2 - x_3 = 0$$

$$\frac{x_1}{-1+1} = \frac{x_2}{1-1} = \frac{x_3}{1-1}$$

$$\frac{x_1}{-1+1} = \frac{x_2}{1-1} = \frac{x_3}{1-1}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_2 \\ 0 \\ x_3 \end{bmatrix}$$

$$\|x\| = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

Case 2:-

$$\text{If } \lambda = 3$$

$$\text{let } x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_1 + 2x_2 - x_3 = 0$$

$$x_1 - x_2 - 0 = 0$$

$$\frac{x_1}{0-1} = \frac{-x_2}{0+1} = \frac{x_3}{1-2}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_2 \\ -x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

Case 3:- $\lambda = 6$

$$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix}$$

$$-3x_1 - x_2 + x_3 = 0$$

$$-x_1 - x_2 - x_3 = 0$$

$$\frac{x_1}{1+1} = \frac{-x_2}{3+i} = \frac{x_3}{3-i}$$

$$\frac{x_1}{2} = \frac{x_2}{3+i} = \frac{x_3}{3-i}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad \|x_3\| = \begin{bmatrix} 1/\sqrt{6} \\ -2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$$

Modal Matrix

$$N = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & -2/\sqrt{6} \\ 1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix}$$

$$N^T = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & -2/\sqrt{6} \\ 1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix}$$

$$N^T A N = D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Canonical form:-

$$\text{Let } y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$Q = y^T A y$$

$$[y_1 \ y_2 \ y_3] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$= 2y_1^2 + 3y_2^2 + 6y_3^2$$

Rank $(s) = 3$

Index $(s) = 3$

Signature $= 2s - r = 2(3) - 3 = 3$

Nature = Positive definite

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$0 = (x + k - p) + [k + k - k - p] = (x + k - p) + [k + k - k - p] = (x + k - p) + [k + k - k - p]$$

$$0 = [x + k - p] + [k + k - k - p] = (x + k - p) + [k + k - k - p]$$

$$0 = [x + k - p] + [k + k - k - p] = (x + k - p) + [k + k - k - p]$$

$$0 = [x + k - p] + [k + k - k - p] = (x + k - p) + [k + k - k - p]$$

$$0 = [x + k - p] + [k + k - k - p] = (x + k - p) + [k + k - k - p]$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} = A$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} = A$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} = A$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} = A$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} = A$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} = A$$

Cayley - Hamilton Theorem

Statement - "Every square matrix satisfies its characteristic equation"

1) verify Cayley Hamilton theorem for the matrix
 $A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ also find its inverse.

Sol The char. Eqⁿ of A is $|A - \lambda I| = 0$
 $\Rightarrow \begin{vmatrix} 2-\lambda & -1 & 0 \\ 3 & 1-\lambda & -1 \\ 1 & 2 & 3-\lambda \end{vmatrix} = 0$

$$\Rightarrow (2-\lambda) [(1-\lambda)(3-\lambda) + 1] + [3(3-\lambda) + 1] = 0$$

$$\Rightarrow (2-\lambda) [\lambda^2 - 4\lambda + 3] + [9 - 3\lambda + 1] = 0$$

$$\Rightarrow (2-\lambda) [\lambda^2 - 4\lambda + 3] + 10 - 3\lambda = 0$$

$$\Rightarrow 2\lambda^2 - 8\lambda + 6 - \lambda^3 + 4\lambda^2 - 3\lambda + 10 - 3\lambda = 0$$

$$\Rightarrow -\lambda^3 + 6\lambda^2 - 14\lambda + 16 = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 14\lambda - 16 = 0 \quad \text{--- (1)}$$

If 'A' satisfies (1) then CHT is verified.
 $\lambda^3 - 6\lambda^2 + 14\lambda - 16 = 0$

$$\begin{bmatrix} -9 & -4 & 6 \\ 0 & -4 & -6 \\ 11 & -12 & 16 \end{bmatrix} = \begin{bmatrix} 6 & -9 & 6 \\ 42 & -12 & -24 \\ 6 & -9 & 16 \end{bmatrix} + \begin{bmatrix} 38 & -14 & 0 \\ 42 & 14 & -14 \\ 28 & 0 & 42 \end{bmatrix} = \begin{bmatrix} 12 & 0 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

$\therefore$ CHT verified.

$$A^3 - 6A^2 + 14A - 17I = 0.$$

Multiply A^{-1} on E-S

$$A^3 A^{-1} - 6A^2 A^{-1} + 14A A^{-1} - 17I A^{-1} = 0$$

$$A^2 - 6A + 14I - 17A^{-1} = 0.$$

$$A^2 - 6A + 14I = 17A^{-1}$$

$$A^{-1} = \frac{1}{17} \begin{bmatrix} 1 & -3 & 1 \\ 2 & -2 & -4 \\ 10 & -2 & 9 \end{bmatrix} = \begin{bmatrix} 12 & -6 & 6 \\ 18 & 6 & -6 \\ 12 & 0 & 12 \end{bmatrix} + \begin{bmatrix} 14 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 14 \end{bmatrix}$$

$$A^{-1} = \frac{3}{17}$$

9) $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

2a). The char. eqⁿ of A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)[(2-\lambda)(2-\lambda)-1] + 1[-1(2-\lambda)+1] + 1(1-2-\lambda) = 0$$

$$\Rightarrow (2-\lambda)[4-2\lambda-2\lambda+\lambda^2-1] + 1[-2+\lambda+1] + 1-2-\lambda = 0$$

$$\Rightarrow (2-\lambda)[\lambda^2-4\lambda+3] + 1-2-\lambda = 0$$

$$\Rightarrow 2\lambda^2-8\lambda+6-\lambda^2+4\lambda-3+1-2-\lambda = 0$$

$$\Rightarrow -\lambda^2+6\lambda-4+4 = 0$$

$$\Rightarrow \lambda^2-6\lambda+4 = 0$$

①

$$A^3 - 6A^2 + 9A - 4I = 0$$

$$\begin{bmatrix} 2 & 2 & -2 & 2 \\ -2 & 2 & -2 & 2 \\ 2 & 2 & -2 & 2 \end{bmatrix} - 6 \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & -2 & 2 \\ -2 & 2 & -2 & 2 \\ 2 & 2 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ 9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} = \begin{bmatrix} 54 & -39 & 39 \\ -39 & 54 & -39 \\ 39 & -39 & 54 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^3 - 6A^2 + 9A - 4I = 0$$

Multiply A^{-1} on B.S

$$A^3 A^{-1} - 6A^2 A^{-1} + 9A A^{-1} - 4I A^{-1} = 0$$

$$A^2 - 6A + 9I - 4A^{-1} = 0$$

$$A^2 - 6A + 9I = 4A^{-1}$$

$$A^{-1} = \frac{1}{4} \left[\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right]$$

$$A^{-1} = \frac{1}{4} \left[\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \right]$$

$$A^{-1} = \begin{bmatrix} 3/4 & 1/4 & -1/4 \\ 1/4 & 3/4 & 1/4 \\ -1/4 & 1/4 & 3/4 \end{bmatrix}$$